

# Decision Making

---

## Basic techniques

## Limitations of Models

---

They

- may be expensive and time-consuming to develop and test
- are often misused and misunderstood (and feared) because of their mathematical and logical complexity
- tend to downplay the role and value of nonquantifiable information
- often have assumptions that oversimplify the variables of the real world

## The Decision-Making Process



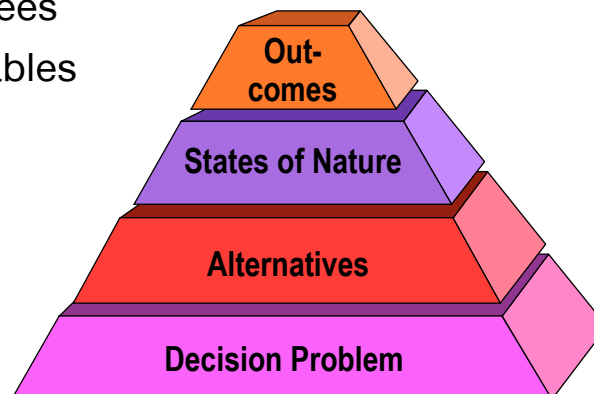
Jamshid Nazemi

*Multi-Criteria Decision Making*

Slide no.: 3

## Ways of Displaying a Decision Problem

- Decision trees
- Decision tables



Jamshid Nazemi

*Multi-Criteria Decision Making*

Slide no.: 4

## Fundamentals of Decision Theory

---

The three types of decision models:

- Decision making under uncertainty
- Decision making under risk
- Decision making under certainty

## Fundamentals of Decision Theory - continued

---

### Terms:

- **Alternative:** course of action or choice
- **State of nature:** an occurrence over which the decision maker has no control

### Symbols used in decision tree:

- A **decision node** from which one of several alternatives may be selected
- A **state of nature node** out of which one state of nature will occur

## Decision Table

---

Alternatives	States of Nature	
	State 1	State 2
Alternative 1	Outcome 1	Outcome 2
Alternative 2	Outcome 3	Outcome 4

Jamshid Nazemi

*Multi-Criteria Decision Making*

Slide no.: 7

## Decision Making Under Risk

---

- Probabilistic decision situation
- States of nature have probabilities of occurrence
- Select alternative with largest expected monetary value (EMV)
  - $EMV = \text{Average return for alternative if decision were repeated many times}$

Jamshid Nazemi

*Multi-Criteria Decision Making*

Slide no.: 8

## Expected Monetary Value Equation

$$EMV (A_i) = \sum_{j=1}^N V_j * P(V_j)$$

Number of states of nature

Value of Payoff

Probability of payoff

$$= V_1 * P(V_1) + V_2 * P(V_2) + \dots + V_N * P(V_N)$$

Alternative  $i$

Jamshid Nazemi

*Multi-Criteria Decision Making*

Slide no.: 9

## Decision Making Under Uncertainty

- **Maximax** - Choose the alternative that maximizes the maximum outcome for every alternative (Optimistic criterion)
- **Maximin** - Choose the alternative that maximizes the minimum outcome for every alternative (Pessimistic criterion)
- **Equally likely** - chose the alternative with the highest average outcome.

Jamshid Nazemi

*Multi-Criteria Decision Making*

Slide no.: 10

## Example - Decision Making Under Uncertainty

Alternatives	States of Nature		Maximum in Row	Minimum in Row	Row Average
	Favorable Market	Unfavorable Market			
Construct large plant	\$200,000	-\$180,000	\$200,000	-\$180,000	\$10,000
Construct small plant	\$100,000	-\$20,000	\$100,000	-\$20,000	\$40,000
	\$0	\$0	\$0	\$0	\$0

Maximax      Maximin      Equally likely

Jamshid Nazemi

*Multi-Criteria Decision Making*

Slide no.: 11

## Example - Decision Making Under Uncertainty

Alternatives	States of Nature		Expected value
	Favorable Market P(0.5)	Unfavorable Market P(0.5)	
Construct large plant	\$200,000	-\$180,000	\$10,000
Construct small plant	\$100,000	-\$20,000	\$40,000
Do nothing	\$0	\$0	\$0

Best choice

Jamshid Nazemi

*Multi-Criteria Decision Making*

Slide no.: 12

## Decision under uncertainty

- Laplace Criterion
- Minimax Criterion
- Savage ( Regret) Criterion
- Hurwicz Criterion

Probabilities	0	0	0	0		
Option 1	0	3	10	10	0	10
Option 2	3	0	0	8	0	8
Option 3	16	11	4	6	4	16
Option 4	25	15	11	0	0	25
			minimum		0	8
					minimin	minimax

Probabilities	0	0	0	0	
Option 1	0	3	10	10	10
Option 2	3	0	0	8	8
Option 3	16	11	4	6	16
Option 4	25	15	11	0	25
Minimax regret					8

Probabilities	0	0	0	0			
Option 1	0	3	10	10	0	10	5
Option 2	3	0	0	8	0	8	4
Option 3	16	11	4	6	4	16	10
Option 4	25	15	11	0	0	25	12.5
				maximum 4		25	12.5
					maximin	maximax	Best Hurwicz

Alfa=.5

هو الحق

## Extreme analysis



## Expected Value of Perfect Information ( $EVPI$ )

- $EVPI$  places an upper bound on what one would pay for additional information
- $EVPI$  is the expected value with perfect information minus the maximum EMV

## Expected Value With Perfect Information ( $EV|PI$ )

$$EV|PI = \sum_{j=1}^n (\text{Best outcome for the state of nature } j) * P(S_j)$$

where  $j=1$  to the number of states of nature,  $n$

## Expected Value of Perfect Information

➤  $EVPI = EV|PI - \text{maximum EMV}$

## Expected Value of Perfect Information

Alternative	State of Nature		EMV
	Favorable Market (\$)	Unfavorable Market (\$)	
Construct a large plant	200,000	-\$180,000	\$20,000
Construct a small plant	\$100,000	\$20,000	\$40,000
Do nothing	\$0	\$0	\$0
<b>Probabilities</b>	0.50	0.50	

## Expected Value of Perfect Information

$EVPI = \text{expected value with perfect information} - \max(\text{EMV})$

$$= \$200,000 * 0.50 + 0 * 0.50 - \$40,000$$

$$= \$60,000$$

## Expected Opportunity Loss

- EOL is the cost of not picking the best solution
- EOL = Expected Regret

## Computing EOL - The - Opportunity Loss Table

Alternative	State of Nature	
	Favorable Market (\$)	Unfavorable Market (\$)
Large Plant	200,000 - 200,000	0 - (-180,000)
Small Plant	200,000 - 100,000	0 - (-20,000)
Do Nothing	200,000 - 0	0-0
Probabilities	0.50	0.50

Jamshid Nazemi

*Multi-Criteria Decision Making*

Slide no.: 23

## The Opportunity Loss Table - continued

Alternative	State of Nature	
	Favorable Market (\$)	Unfavorable Market (\$)
Large Plant	0	180,000
Small Plant	100,000	20,000
Do Nothing	200,000	0
Probabilities	0.50	0.50

Jamshid Nazemi

*Multi-Criteria Decision Making*

Slide no.: 24

## The Opportunity Loss Table - continued

Alternative		EOL
Large Plant	$(0.50)*\$0 +$ $(0.50)*(\$180,000)$	\$90,000
Small Plant	$(0.50)*(\$100,000)$ $+ (0.50)*(\$20,000)$	\$60,000
Do Nothing	$(0.50)*(\$200,000)$ $+ (0.50)*(\$0)$	\$100,000

Jamshid Nazemi

*Multi-Criteria Decision Making*

Slide no.: 25

## Sensitivity Analysis

$$\text{EMV}(\text{Large Plant}) = \$200,000P - (1-P)\$180,000$$

$$\text{EMV}(\text{Small Plant}) = \$100,000P - \$20,000(1-P)$$

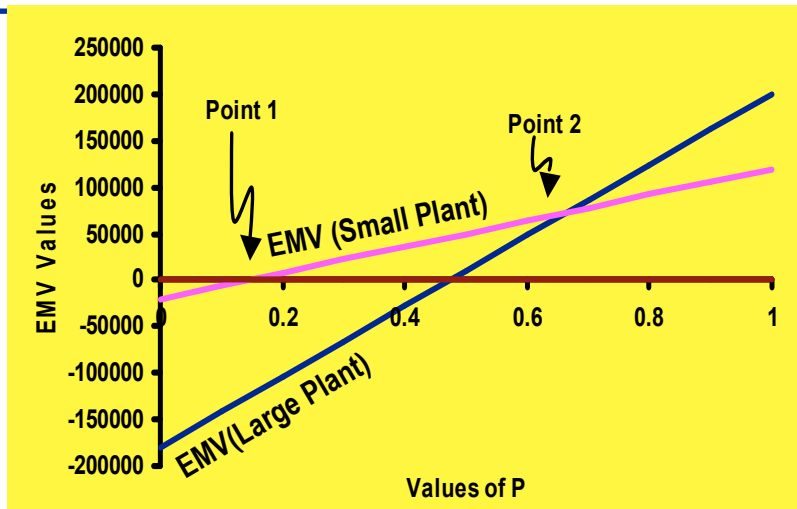
$$\text{EMV}(\text{Do Nothing}) = \$0P + 0(1-P)$$

Jamshid Nazemi

*Multi-Criteria Decision Making*

Slide no.: 26

## Sensitivity Analysis - continued



Jamshid Nazemi

*Multi-Criteria Decision Making*

Slide no.: 27

## Decision Trees

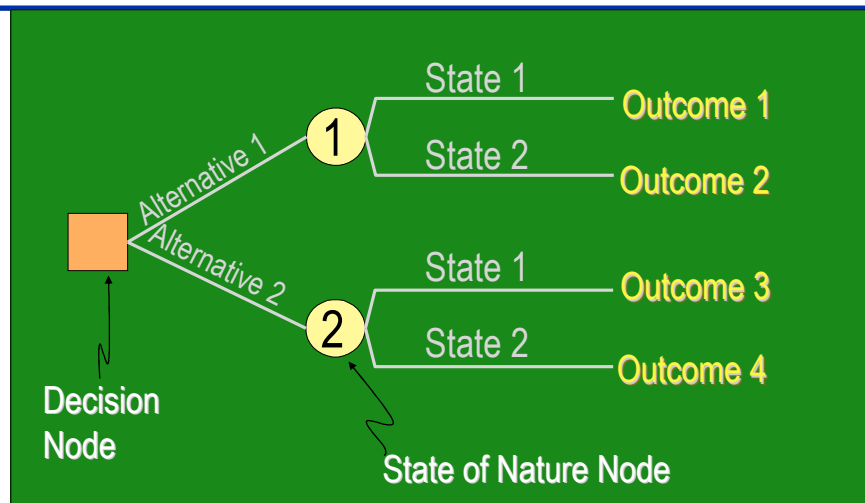
- Graphical display of decision process
- Used for solving problems
  - With 1 set of alternatives and states of nature, decision tables can be used also
  - With several sets of alternatives and states of nature (sequential decisions), decision tables cannot be used
- EMV is criterion most often used

Jamshid Nazemi

*Multi-Criteria Decision Making*

Slide no.: 28

## Decision Tree



Jamshid Nazemi

*Multi-Criteria Decision Making*

Slide no.: 29

## Analyzing Problems with Decision Trees

- Define the problem
- Structure or draw the decision tree
- Assign probabilities to the states of nature
- Estimate payoffs for each possible combination of alternatives and states of nature
- Solve the problem by computing expected monetary values for each state-of-nature node

Jamshid Nazemi

*Multi-Criteria Decision Making*

Slide no.: 30

# Game theory

## Basic payoff tables

## مثال - تئوری بازیها ( استراتژی شیر یا خط )

	Col strat1	Col strat2	Row Minimum	Maximin
Row strategy 1	1.	-1.	-1.	-1.
Row strategy 2	-1.	1.	-1.	-1.
Column Maximum	1.	1.		
Minimax	1.	1.		
$-1 \leq \text{value} \leq 1$				



## مثال - بازی دو نفره دارای نقطه زینی Saddle point

Game Theory Results					
	Col strat1	Col strat2	Col strat3	Col strat4	Row Mix
1	8.	2.	9.	5.	0.
2	6.	5.	7.	18.	1.
3	7.	3.	-4.	10.	0.
Column Mix-->	0.	1.	0.	0.	
Value of game (to row)	5.				

Maximin & Minimax						
	Col strat1	Col strat2	Col strat3	Col strat4	Row Minimum	Maximin
1	8.	2.	9.	5.	2.	
2	6.	5.	7.	18.	5.	5.
3	7.	3.	-4.	10.	-4.	
Column Maximum	8.	5.	9.	18.		
Minimax		5.				
Value= 5						

Jamshid Nazemi

Multi-Criteria Decision Making

Slide no.: 33

## مسئله بازی دو نفره - عدم تعادل

Game Theory Results					
	Col strat1	Col strat2	Col strat3	Col strat4	Row Mix
Row strategy 1	5.	-10.	9.	0.	0.
Row strategy 2	6.	7.	8.	1.	0.
Row strategy 3	8.	7.	15.	2.	0.2778
Row strategy 4	3.	4.	-1.	4.	0.7222
Column Mix-->	0.	0.	0.1111	0.8889	
Value of game (to row)	3.4444				

Maximin & Minimax						
	Col strat1	Col strat2	Col strat3	Col strat4	Row Minimum	Maximin
Row strategy 1	5.	-10.	9.	0.	-10.	
Row strategy 2	6.	7.	8.	1.	1.	
Row strategy 3	8.	7.	15.	2.	2.	2.
Row strategy 4	3.	4.	-1.	4.	-1.	
Column Maximum	8.	7.	15.	4.		
Minimax				4.		
2 <= value <= 4						

Jamshid Nazemi

Multi-Criteria Decision Making

Slide no.: 34

## Payoff profit matrix

	s1	s2	s3	s4	s5
A1	15	10	0	-6	17
A2	3	14	8	9	2
A3	1	5	14	20	-3
A4	7	19	10	2	0

Jamshid Nazemi

*Multi-Criteria Decision Making*

Slide no.: 35

## Compare the solutions

- Laplace Criterion
- Minimax Criterion
- Savage ( Regret) Criterion
- Hurwicz Criterion (  $\alpha=.5$ )

Jamshid Nazemi

*Multi-Criteria Decision Making*

Slide no.: 36

- Find saddle point
- Value of the game

	B			
A	8	6	2	8
	8	9	4	5
	7	5	3	5

- Find saddle point
- Value of the game

	B			
A	4	-4	-5	6
	-3	-4	-9	-2
	6	7	-8	-9
	7	3	-9	5

- Find a range for P & Q for (2,2) saddle point game

	B			
	1	P	3	
A	Q	5	10	
	6	2	3	

Game Theory Results					
	Col strat1	Col strat2	Col strat3	Col strat4	Row Mix
Row strategy 1	8.	6.	2.	8.	0.
Row strategy 2	8.	9.	4.	5.	1.
Row strategy 3	7.	5.	3.	5.	0.
Column Mix-->	0.	0.	1.	0.	
Value of game (to row)	4.				

<untitled>						
	Col strat1	Col strat2	Col strat3	Col strat4	Row Minimum	Maximin
Row strategy 1	8.	6.	2.	8.	2.	
Row strategy 2	8.	9.	4.	5.	4.	4.
Row strategy 3	7.	5.	3.	5.	3.	
Column Maximum	8.	9.	4.	8.		
Minimax			4.			
Value= 4						

**Game Theory Results**

	Col strat1	Col strat2	Col strat3	Col strat4	Row Mix
Row strategy 1	4.	-4.	-5.	6.	1.
Row strategy 2	-3.	-4.	-9.	-2.	0.
Row strategy 3	6.	7.	-8.	-9.	0.
Row strategy 4	7.	3.	-9.	5.	0.
Column Mix-->	0.	0.	1.	0.	
Value of game (to row)	-5.				

**Maximin & Minimax**

	Col strat1	Col strat2	Col strat3	Col strat4	Row Minimum	Maximin
Row strategy 1	4.	-4.	-5.	6.	-5.	5.
Row strategy 2	-3.	-4.	-9.	-2.	-9.	
Row strategy 3	6.	7.	-8.	-9.	-9.	
Row strategy 4	7.	3.	-9.	5.	-9.	
Column Maximum	7.	7.	-5.	6.		
Minimax			-5.			
Value=-5						

# Decision Making

---

## Basic techniques

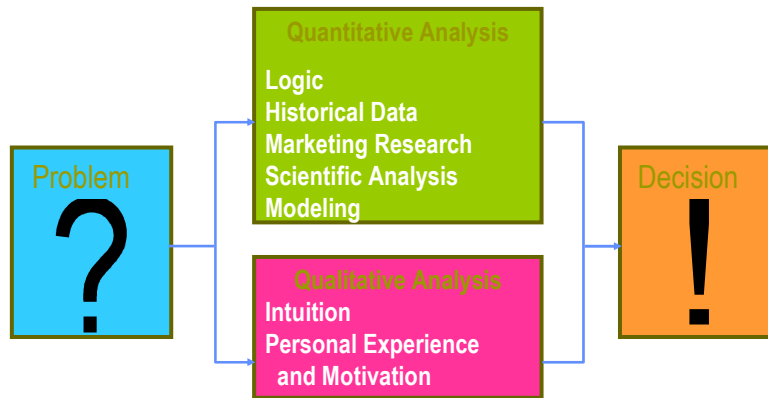
## Limitations of Models

---

They

- may be expensive and time-consuming to develop and test
- are often misused and misunderstood (and feared) because of their mathematical and logical complexity
- tend to downplay the role and value of nonquantifiable information
- often have assumptions that oversimplify the variables of the real world

## The Decision-Making Process



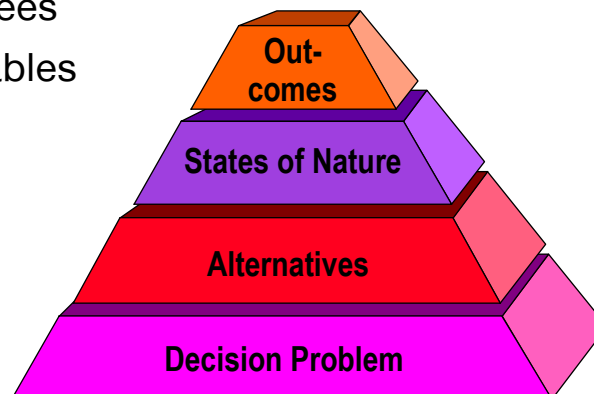
Jamshid Nazemi

*Multi-Criteria Decision Making*

Slide no.: 3

## Ways of Displaying a Decision Problem

- Decision trees
- Decision tables



Jamshid Nazemi

*Multi-Criteria Decision Making*

Slide no.: 4

## Fundamentals of Decision Theory

---

The three types of decision models:

- Decision making under uncertainty
- Decision making under risk
- Decision making under certainty

## Fundamentals of Decision Theory - continued

---

### Terms:

- **Alternative:** course of action or choice
- **State of nature:** an occurrence over which the decision maker has no control

### Symbols used in decision tree:

- A **decision node** from which one of several alternatives may be selected
- A **state of nature node** out of which one state of nature will occur



## Decision Table

---

Alternatives	States of Nature	
	State 1	State 2
Alternative 1	Outcome 1	Outcome 2
Alternative 2	Outcome 3	Outcome 4

Jamshid Nazemi

*Multi-Criteria Decision Making*

Slide no.: 7

## Decision Making Under Risk

---

- Probabilistic decision situation
- States of nature have probabilities of occurrence
- Select alternative with largest expected monetary value (EMV)
  - EMV = Average return for alternative if decision were repeated many times

Jamshid Nazemi

*Multi-Criteria Decision Making*

Slide no.: 8

## Expected Monetary Value Equation

$$EMV (A_i) = \sum_{j=1}^N V_j * P(V_j)$$

Number of states of nature

Value of Payoff

Probability of payoff

$$= V_1 * P(V_1) + V_2 * P(V_2) + \dots + V_N * P(V_N)$$

Alternative  $i$

Jamshid Nazemi

*Multi-Criteria Decision Making*

Slide no.: 9

## Decision Making Under Uncertainty

- **Maximax** - Choose the alternative that maximizes the maximum outcome for every alternative (Optimistic criterion)
- **Maximin** - Choose the alternative that maximizes the minimum outcome for every alternative (Pessimistic criterion)
- **Equally likely** - chose the alternative with the highest average outcome.

Jamshid Nazemi

*Multi-Criteria Decision Making*

Slide no.: 10

## Example - Decision Making Under Uncertainty

Alternatives	States of Nature		Maximum in Row	Minimum in Row	Row Average
	Favorable Market	Unfavorable Market			
Construct large plant	\$200,000	-\$180,000	\$200,000	-\$180,000	\$10,000
Construct small plant	\$100,000	-\$20,000	\$100,000	-\$20,000	\$40,000
	\$0	\$0	\$0	\$0	\$0

Maximax      Maximin      Equally likely

Jamshid Nazemi

*Multi-Criteria Decision Making*

Slide no.: 11

## Example - Decision Making Under Uncertainty

Alternatives	States of Nature		Expected value
	Favorable Market P(0.5)	Unfavorable Market P(0.5)	
Construct large plant	\$200,000	-\$180,000	\$10,000
Construct small plant	\$100,000	-\$20,000	\$40,000
Do nothing	\$0	\$0	\$0

Best choice

Jamshid Nazemi

*Multi-Criteria Decision Making*

Slide no.: 12

## Decision under uncertainty

---

- Laplace Criterion
- Minimax Criterion
- Savage ( Regret) Criterion
- Hurwicz Criterion

Probabilities	0	0	0	0		
Option 1	0	3	10	10	0	10
Option 2	3	0	0	8	0	8
Option 3	16	11	4	6	4	16
Option 4	25	15	11	0	0	25
			minimum		0	8
					minimin	minimax

Probabilities	0	0	0	0	
Option 1	0	3	10	10	10
Option 2	3	0	0	8	8
Option 3	16	11	4	6	16
Option 4	25	15	11	0	25
Minimax regret					8

Probabilities	0	0	0	0			
Option 1	0	3	10	10	0	10	5
Option 2	3	0	0	8	0	8	4
Option 3	16	11	4	6	4	16	10
Option 4	25	15	11	0	0	25	12.5
				maximum 4		25	12.5
					maximin	maximax	Best Hurwicz

Alfa=.5

مدرس تربیت  
هوالمع

## Extreme analysis

## Expected Value of Perfect Information ( $EVPI$ )

➤  $EVPI$  places an upper bound on what one would pay for additional information

➤  $EVPI$  is the expected value with perfect information minus the maximum EMV

## Expected Value With Perfect Information ( $EV|PI$ )

$$EV|PI = \sum_{j=1}^n (\text{Best outcome for the state of nature } j) * P(S_j)$$

where  $j=1$  to the number of states of nature,  $n$

## Expected Value of Perfect Information

➤  $EVPI = EV|PI - \text{maximum EMV}$

## Expected Value of Perfect Information

Alternative	State of Nature		EMV
	Favorable Market (\$)	Unfavorable Market (\$)	
Construct a large plant	200,000	-\$180,000	\$20,000
Construct a small plant	\$100,000	\$20,000	\$40,000
Do nothing	\$0	\$0	\$0
<b>Probabilities</b>	0.50	0.50	

## Expected Value of Perfect Information

$EVPI = \text{expected value with perfect information} - \max(\text{EMV})$

$$= \$200,000 * 0.50 + 0 * 0.50 - \$40,000$$

$$= \$60,000$$

## Expected Opportunity Loss

- EOL is the cost of not picking the best solution
- EOL = Expected Regret



## Computing EOL - The - Opportunity Loss Table

Alternative	State of Nature	
	Favorable Market (\$)	Unfavorable Market (\$)
Large Plant	200,000 - 200,000	0 - (-180,000)
Small Plant	200,000 - 100,000	0 - (-20,000)
Do Nothing	200,000 - 0	0-0
Probabilities	0.50	0.50

Jamshid Nazemi

*Multi-Criteria Decision Making*

Slide no.: 23

## The Opportunity Loss Table - continued

Alternative	State of Nature	
	Favorable Market (\$)	Unfavorable Market (\$)
Large Plant	0	180,000
Small Plant	100,000	20,000
Do Nothing	200,000	0
Probabilities	0.50	0.50

Jamshid Nazemi

*Multi-Criteria Decision Making*

Slide no.: 24

## The Opportunity Loss Table - continued

Alternative		EOL
Large Plant	$(0.50)*\$0 +$ $(0.50)*(\$180,000)$	\$90,000
Small Plant	$(0.50)*(\$100,000)$ $+ (0.50)*(\$20,000)$	\$60,000
Do Nothing	$(0.50)*(\$200,000)$ $+ (0.50)*(\$0)$	\$100,000

Jamshid Nazemi

*Multi-Criteria Decision Making*

Slide no.: 25

## Sensitivity Analysis

$$\text{EMV}(\text{Large Plant}) = \$200,000P - (1-P)\$180,000$$

$$\text{EMV}(\text{Small Plant}) = \$100,000P - \$20,000(1-P)$$

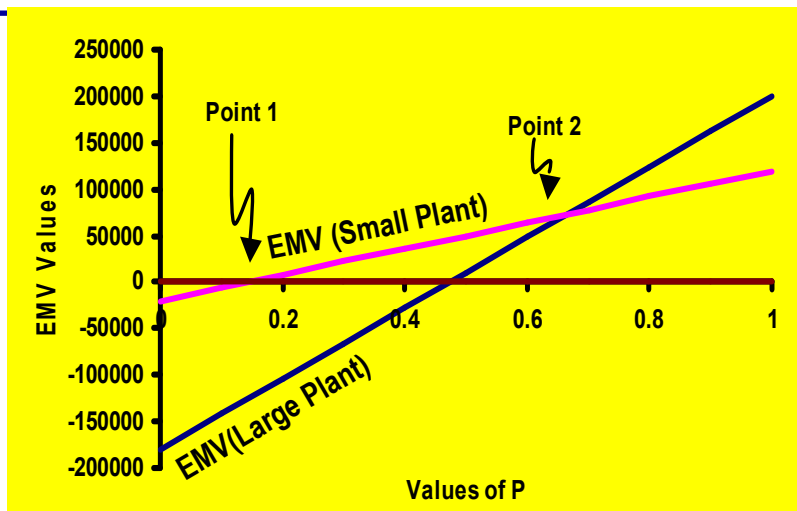
$$\text{EMV}(\text{Do Nothing}) = \$0P + 0(1-P)$$

Jamshid Nazemi

*Multi-Criteria Decision Making*

Slide no.: 26

## Sensitivity Analysis - continued



Jamshid Nazemi

*Multi-Criteria Decision Making*

Slide no.: 27

## Decision Trees

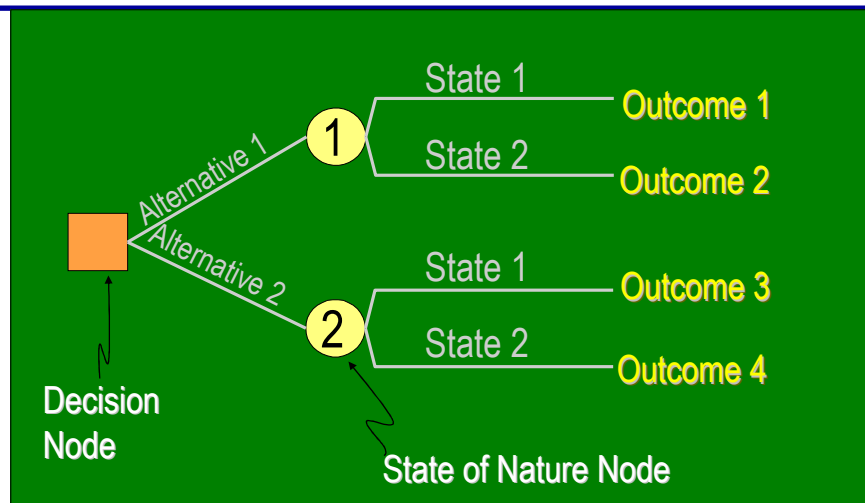
- Graphical display of decision process
- Used for solving problems
  - With 1 set of alternatives and states of nature, decision tables can be used also
  - With several sets of alternatives and states of nature (sequential decisions), decision tables cannot be used
- EMV is criterion most often used

Jamshid Nazemi

*Multi-Criteria Decision Making*

Slide no.: 28

## Decision Tree



Jamshid Nazemi

*Multi-Criteria Decision Making*

Slide no.: 29

## Analyzing Problems with Decision Trees

- Define the problem
- Structure or draw the decision tree
- Assign probabilities to the states of nature
- Estimate payoffs for each possible combination of alternatives and states of nature
- Solve the problem by computing expected monetary values for each state-of-nature node

Jamshid Nazemi

*Multi-Criteria Decision Making*

Slide no.: 30

# Game theory

## Basic payoff tables

## مثال - تئوری بازیها ( استراتژی شیر یا خط )

	Col strat1	Col strat2	Row Minimum	Maximin
Row strategy 1	1.	-1.	-1.	-1.
Row strategy 2	-1.	1.	-1.	-1.
Column Maximum	1.	1.		
Minimax	1.	1.		
$-1 \leq \text{value} \leq 1$				

## مثال - بازی دو نفره دارای نقطه زینی Saddle point

Game Theory Results					
	Col strat1	Col strat2	Col strat3	Col strat4	Row Mix
1	8.	2.	9.	5.	0.
2	6.	5.	7.	18.	1.
3	7.	3.	-4.	10.	0.
Column Mix-->	0.	1.	0.	0.	
Value of game (to row)	5.				

Maximin & Minimax						
	Col strat1	Col strat2	Col strat3	Col strat4	Row Minimum	Maximin
1	8.	2.	9.	5.	2.	
2	6.	5.	7.	18.	5.	5.
3	7.	3.	-4.	10.	-4.	
Column Maximum	8.	5.	9.	18.		
Minimax		5.				
Value= 5						

Jamshid Nazemi

Multi-Criteria Decision Making

Slide no.: 33

## مسئله بازی دو نفره - عدم تعادل

Game Theory Results					
	Col strat1	Col strat2	Col strat3	Col strat4	Row Mix
Row strategy 1	5.	-10.	9.	0.	0.
Row strategy 2	6.	7.	8.	1.	0.
Row strategy 3	8.	7.	15.	2.	0.2778
Row strategy 4	3.	4.	-1.	4.	0.7222
Column Mix-->	0.	0.	0.1111	0.8889	
Value of game (to row)	3.4444				

Maximin & Minimax						
	Col strat1	Col strat2	Col strat3	Col strat4	Row Minimum	Maximin
Row strategy 1	5.	-10.	9.	0.	-10.	
Row strategy 2	6.	7.	8.	1.	1.	
Row strategy 3	8.	7.	15.	2.	2.	2.
Row strategy 4	3.	4.	-1.	4.	-1.	
Column Maximum	8.	7.	15.	4.		
Minimax				4.		
2 <= value <= 4						

Jamshid Nazemi

Multi-Criteria Decision Making

Slide no.: 34

## Payoff profit matrix

	s1	s2	s3	s4	s5
A1	15	10	0	-6	17
A2	3	14	8	9	2
A3	1	5	14	20	-3
A4	7	19	10	2	0

Jamshid Nazemi

*Multi-Criteria Decision Making*

Slide no.: 35

## Compare the solutions

- Laplace Criterion
- Minimax Criterion
- Savage ( Regret) Criterion
- Hurwicz Criterion (  $\alpha=.5$ )

Jamshid Nazemi

*Multi-Criteria Decision Making*

Slide no.: 36

- Find saddle point
- Value of the game

	B			
A	8	6	2	8
	8	9	4	5
	7	5	3	5

- Find saddle point
- Value of the game

	B			
A	4	-4	-5	6
	-3	-4	-9	-2
	6	7	-8	-9
	7	3	-9	5



➤ Find a range for P & Q for (2,2) saddle point game

	B			
	1	P	3	
A	Q	5	10	
	6	2	3	