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Fuzzy MCDM

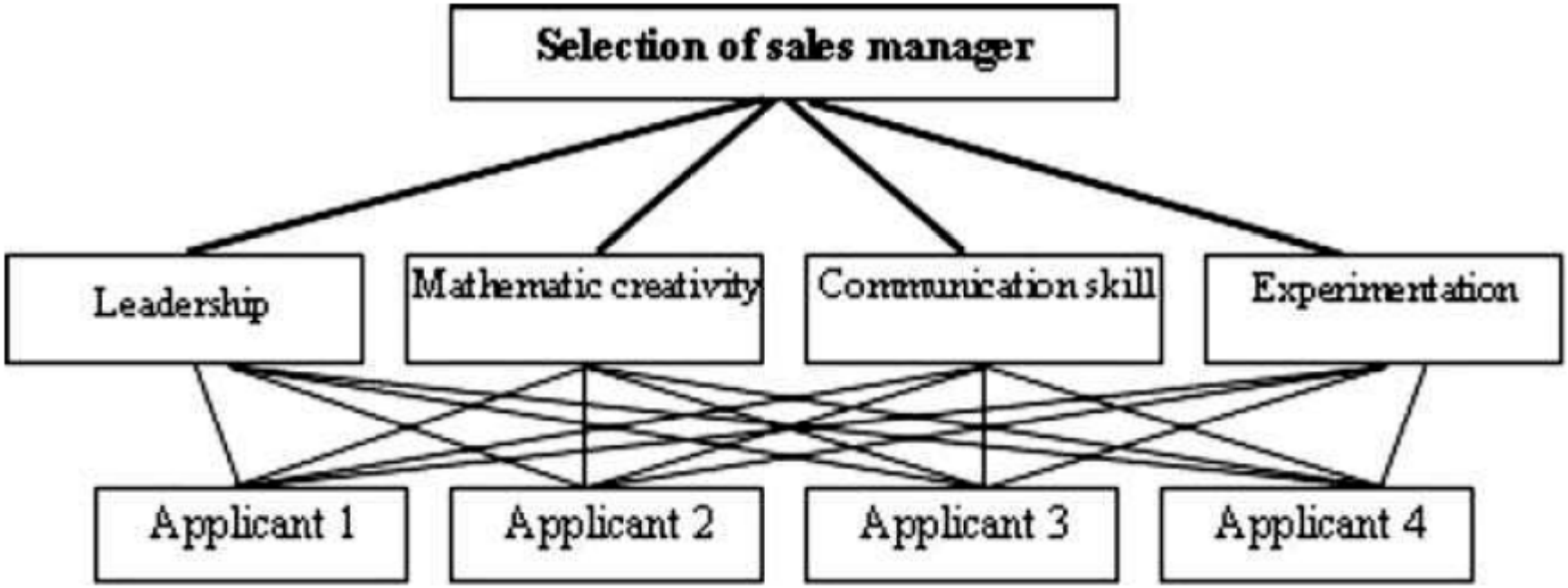

Figure 1. The hierarchy of the sales manager selection

Table 2. Pair-Wise Comparisons of Applicants for Leadership

	A_1	A_2	A_3	A_4
A_1	(1, 1, 1)	(2/5, 1/2, 2/3) (2/5, 1/2, 2/3)	(2/3, 1, 3/2)	(2/9, 1/4, 2/7) (2/9, 1/4, 2/7) (2/7, 1/3, 2/5)
A_2	(3/2, 2, 5/2) (3/2, 2, 5/2)	(1, 1, 1)	(5/2, 3, 7/2) (3/2, 2, 5/2)	(2/5, 1/2, 2/3) (2/3, 1, 3/2) (2/3, 1, 3/2)
A_3	(2/3, 1, 3/2)	(2/7, 1/3, 2/5) (2/5, 1/2, 2/3)	(1, 1, 1)	(2/5, 1/2, 2/3) (2/3, 1, 3/2)
A_4	(7/2, 4, 9/2) (7/2, 4, 9/2) (5/2, 3, 7/2)	(3/2, 2, 5/2) (2/3, 1, 3/2) (2/3, 1, 3/2)	(3/2, 2, 5/2) (2/3, 1, 3/2)	(1, 1, 1)

Table 3. Pair-Wise Comparisons of Applicants for Mathematic Creativity

	A_1	A_2	A_3	A_4
A_1	(1, 1, 1)	(3/2, 2, 5/2) (3/2, 2, 5/2)	(2/7, 1/3, 2/5) (2/9, 1/4, 2/7) (2/9, 1/4, 2/7)	(2/3, 1, 3/2)
A_2	(2/5, 1/2, 2/3) (2/5, 1/2, 2/3)	(1, 1, 1)	(2/5, 1/2, 2/3)	(2/5, 1/2, 2/3) (2/3, 1, 3/2)
A_3	(5/2, 3, 7/2) (7/2, 4, 9/2) (7/2, 4, 9/2)	(3/2, 2, 5/2)	(1, 1, 1)	(5/2, 3, 7/2) (5/2, 3, 7/2) (3/2, 2, 5/2)
A_4	(2/3, 1, 3/2)	(3/2, 2, 5/2) (2/3, 1, 3/2)	(2/7, 1/3, 2/5) (2/7, 1/3, 2/5) (2/5, 1/2, 2/3)	(1, 1, 1)

Table 4. Pair-Wise Comparisons of Applicants for Communication Skill

	A_1	A_2	A_3	A_4
A_1	(1, 1, 1)	(5/2, 3, 7/2) (5/2, 3, 7/2)	(5/2, 3, 7/2)	(2/3, 1, 3/2)
A_2	(2/7, 1/3, 2/5) (2/7, 1/3, 2/5)	(1, 1, 1)	(2/3, 1, 3/2)	(3/2, 2, 5/2) (3/2, 2, 5/2)
A_3	(2/7, 1/3, 2/5)	(2/3, 1, 3/2)	(1, 1, 1)	(2/7, 1/3, 2/5)
A_4	(2/3, 1, 3/2)	(2/5, 1/2, 2/3) (2/5, 1/2, 2/3)	(5/2, 3, 7/2)	(1, 1, 1)

Table 5. Pair-Wise Comparisons of Applicants for Experimentation

	A_1	A_2	A_3	A_4
A_1	(1, 1, 1)	(2/3, 1, 3/2)	(7/2, 4, 9/2) (5/2, 3, 7/2)	(2/5, 1/2, 2/3)
A_2	(2/3, 1, 3/2)	(1, 1, 1)	(3/2, 2, 5/2) (5/2, 3, 7/2) (5/2, 3, 7/2)	(2/3, 1, 3/2)
A_3	(2/9, 1/4, 2/7) (2/7, 1/3, 2/5)	(2/5, 1/2, 2/3) (2/7, 1/3, 2/5) (2/7, 1/3, 2/5)	(1, 1, 1)	(2/7, 1/3, 2/5)
A_4	(3/2, 2, 5/2)	(2/3, 1, 3/2)	(5/2, 3, 7/2)	(1, 1, 1)

Table 6. Pair-Wise Comparisons of Attributes

	X_1	X_2	X_3	X_4
X_1	(1, 1, 1)	(3/2, 2, 5/2) (3/2, 2, 5/2) (2/3, 1, 3/2)	(7/2, 4, 9/2)	(2/3, 1, 3/2)
X_2	(2/5, 1/2, 2/3) (2/5, 1/2, 2/3) (2/3, 1, 3/2)	(1, 1, 1)	(3/2, 2, 5/2)	(2/5, 1/2, 2/3) (2/3, 1, 3/2)
X_3	(2/9, 1/4, 2/7)	(2/5, 1/2, 2/3)	(1, 1, 1)	(2/9, 1/4, 2/7)
X_4	(2/3, 1, 3/2)	(3/2, 2, 5/2) (2/3, 1, 3/2)	(7/2, 4, 9/2)	(1, 1, 1)

$$l_1\left(\sum_{j=2}^{4} P_{1j}\right) - \sum_{j=2}^{4} P_{1j}u_j = \sum_{j=2}^{4}\sum_{k=1}^{P_{ij}} \ln\left(l_{1jk}\right)$$

$$l_1\left(P_{12}+P_{13}+P_{14}\right)-\left(P_{12}u_2+P_{13}u_3+P_{14}u_4\right)=\sum_{k=1}^{P_{12}}\ln\left(l_{12k}\right)+\sum_{k=1}^{P_{13}}\ln\left(l_{13k}\right)+\sum_{k=1}^{P_{14}}\ln\left(l_{14k}\right)$$

where

$P_{12} = 2$ (two decision makers)
$P_{13} = 1$ (one decision maker)
$P_{14} = 3$ (three decision makers)

$$6l_1 - 2u_2 - u_3 - 3u_4 = \begin{pmatrix} \ln(2/5)+\ln(2/5)+\ln(2/3) \\ +\ln(2/9)+\ln(2/9)+\ln(2/7) \end{pmatrix}$$

$$l_2\left(\sum_{\substack{j=1\\ j\neq 2}}^{4} P_{2j}\right)-\sum_{\substack{j=1\\ j\neq 2}}^{4} P_{2j}u_j=\sum_{\substack{j=1\\ j\neq 2}}^{4}\sum_{k=1}^{P_{ij}}\ln\left(l_{2jk}\right)$$

$$l_2\left(P_{21}+P_{23}+P_{24}\right)-\left(P_{21}u_1+P_{23}u_3+P_{24}u_4\right)=$$
$$\sum_{k=1}^{P_{21}}\ln\left(l_{21k}\right)+\sum_{k=1}^{P_{23}}\ln\left(l_{23k}\right)+\sum_{k=1}^{P_{24}}\ln\left(l_{24k}\right)$$

where

$P_{21}=2$ (two decision makers)
$P_{23}=2$ (two decision makers)
$P_{24}=3$ (three decision makers)

$$7l_2-2u_1-2u_3-3u_4=\begin{pmatrix}\left(\ln(3/2)+\ln(3/2)\right)+\left(\ln(5/2)+\ln(3/2)\right)\\ +\left(\ln(2/5)+\ln(2/3)+\ln(2/3)\right)\end{pmatrix}$$

For all the equations

$$6l_1 - 2u_2 - 1u_3 - 3u_4 = -6.4989$$
$$7l_2 - 2u_1 - 2u_3 - 3u_4 = 0.4054$$
$$5l_3 - 1u_1 - 2u_2 - 2u_4 = -3.8962$$
$$8l_4 - 3u_1 - 3u_2 - 2u_3 = 3.0163$$
$$6m_1 - 2m_2 - 1m_3 - 3m_4 = -5.2574$$
$$7m_2 - 2m_1 - 2m_3 - 3m_4 = 0.4054$$
$$5m_3 - 1m_1 - 2m_2 - 2m_4 = -3.8962$$
$$8m_4 - 3m_1 - 3m_2 - 2m_3 = 3.0163$$
$$6u_1 - 2l_2 - 1l_3 - 3l_4 = -3.8272$$
$$7u_2 - 2l_1 - 2l_3 - 3l_4 = 4.4071$$
$$5u_3 - 1l_1 - 2l_2 - 2l_4 = -0.9162$$
$$8u_4 - 3l_1 - 3l_2 - 2l_3 = 7.3098$$

Table 7. The Solutions to the Equations

I	l_i	m_i	u_i
1	0	0	0.1443
2	0.7870	0.8919	1.1028
3	0.1936	0.2849	0.5216
4	0.9751	1.0629	1.2572

Table 8. The Exponentials of l_i, m_i, and u_i

I	$\exp(l_i)$	$\exp(m_i)$	$\exp(u_i)$
1	1.0000	1.0000	1.1552
2	2.1967	2.4397	3.0125
3	1.2136	1.3296	1.6847
4	2.6514	2.8947	3.5155

$$r_{11} = (\lambda_1 \exp(l_1),\ \lambda_2 \exp(m_1),\ \lambda_3 \exp(u_1))$$

The terms λ_1, λ_2, and λ_3 are computed as

$$\lambda_1 = \left(\sum_{i=1}^{4} \exp(u_i) \right)^{-1} = 0.1067 \ , \lambda_2 = \left(\sum_{i=1}^{4} \exp(m_i) \right)^{-1} = 0.1304$$

$$\lambda_3 = \left(\sum_{i=1}^{4} \exp(l_i) \right)^{-1} = 0.1416 .$$

$$r_{11} = (0.1076, 0.1304, 0.1635)$$
$$r_{12} = (0.2343, 0.3181, 0.4265)$$
$$r_{13} = (0.1294, 0.1733, 0.2385)$$
$$r_{14} = (0.2829, 0.3774, 0.4977).$$

After applying to all matrix's

Table 9. All Results

	X1	X2	X3	X4
A1	(0.1067, 0.1304, 0.1635)	(0.1434, 0.1793, 0.2225)	(0.3498, 0.4363, 0.5362)	(0.2018, 0.2729, 0.3730)
A2	(0.2343, 0.3181, 0.4265)	(0.1035, 0.1288, 0.1680)	(0.1708, 0.2190, 0.2787)	(0.1868, 0.2760, 0.4037)
A3	(0.1295, 0.1733, 0.2385)	(0.4457, 0.5049, 0.5568)	(0.1042, 0.1313, 0.1685)	(0.0855, 0.0974, 0.1139)
A4	(0.2829, 0.3774, 0.4977)	(0.1413, 0.1868, 0.2495)	(0.1641, 0.2130, 0.2827)	(0.2556, 0.3530, 0.4772)

W = [(0.2579, 0.3509, 0.4703), (0.1609, 0.2199, 0.3054), (0.0812, 0.0932, 0.1101), (0.2418, 0.3354, 0.4612)]

$$u_i = \sum_{j=1}^{n} w_j r_{ij}$$

$$U_1 = (0.1277, 0.2173, 0.3759)$$
$$U_2 = (0.1361, 0.2529, 0.4687)$$
$$U_3 = (0.1342, 0.2167, 0.3532)$$
$$U_4 = (0.1708, 0.3117, 0.5614).$$

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Simple additive

example

$$\begin{array}{c} \\ A_1 \\ A_2 \end{array} \begin{array}{c} \begin{array}{cc} X_1 & X_2 \end{array} \\ \begin{bmatrix} \text{excellent} & \text{fair} \\ \text{good} & \text{good} \end{bmatrix} \end{array}$$

$$(w_1, w_2) = (\text{important},\ \text{very important})$$

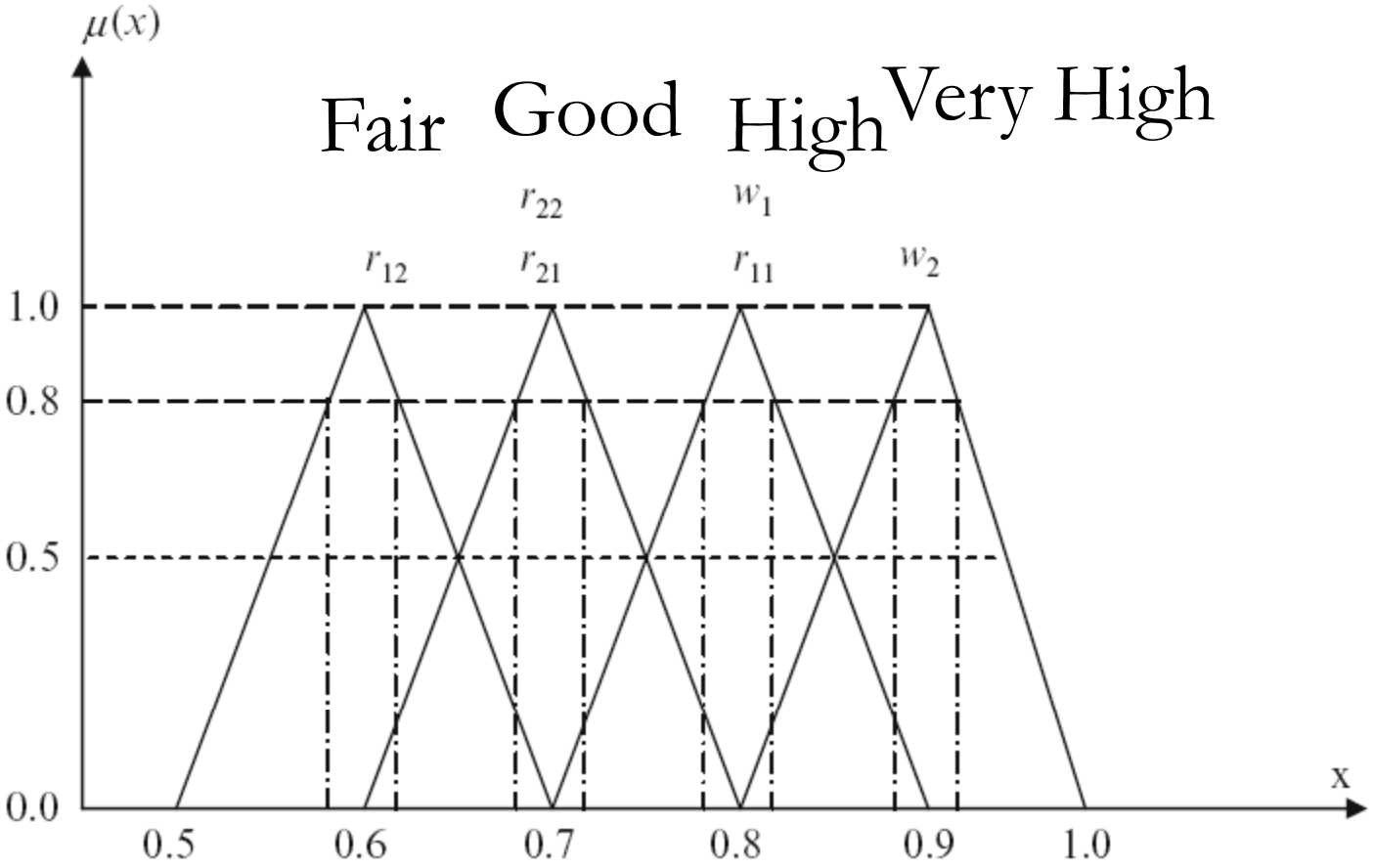


Figure 2. Fuzzy representation of linguistic terms

Step 1. Let's set $\alpha_0 = 0.80$.

Step 2. Let's identify $\hat{x}_{11}, \hat{x}_{12}, \hat{x}_{21}, \hat{x}_{22}, \hat{y}_1, \hat{y}_2$ values such that $\mu_{r11}(\hat{x}_{11}) = \mu_{r12}(\hat{x}_{12}) = \mu_{r21}(\hat{x}_{21}) = \mu_{w1}(\hat{y}_1) = \mu_{w2}(\hat{y}_2) = 0.80$. The values providing this equality are given in the Table 1.

Table 1. $\alpha - cut$ Values While $\alpha_0 = 0.80$

$\hat{x}_{12}$	$\hat{x}_{11}$	$\hat{x}_{21}$	$\hat{x}_{22}$	$\hat{y}_1$	$\hat{y}_2$
0.62	0.82	0.72	0.72	0.82	0.92
0.58	0.78	0.68	0.68	0.78	0.88

$$u_i = \frac{\sum_{j=1}^{n} y_j x_{ij}}{\sum_{j=1}^{n} y_j} \qquad (9)$$

Step 3. There are a total of $2^4 = 16$ possible combinations of $(\hat{x}_{11}, \hat{x}_{12}, \hat{y}_1, \hat{y}_2)$ and $(\hat{x}_{21}, \hat{x}_{22}, \hat{y}_1, \hat{y}_2)$. Using Eq. (9) on all x_{ij} and y_j combinations, we obtain 16 u_1 values. The u_1 values are given in Table 2.

Table 2. Possible Combinations of x_{ij} and y_j and Their Corresponding u_1 Values

x_{11}	x_{12}	y_1	y_2	u_1	
0.82	0.62	0.82	0.92	0.714253	
0.82	0.62	0.82	0.88	0.716471	MAX.
0.82	0.62	0.78	0.92	0.711765	
0.82	0.62	0.78	0.88	0.713976	
0.82	0.58	0.82	0.92	0.693103	
0.82	0.58	0.82	0.88	0.695765	
0.82	0.58	0.78	0.92	0.690118	
0.82	0.58	0.78	0.88	0.692771	
0.78	0.62	0.82	0.92	0.695402	
0.78	0.62	0.82	0.88	0.697176	
0.78	0.62	0.78	0.92	0.693412	
0.78	0.62	0.78	0.88	0.695181	
0.78	0.58	0.82	0.92	0.674253	
0.78	0.58	0.82	0.88	0.676471	
0.78	0.58	0.78	0.92	0.671765	MIN.
0.78	0.58	0.78	0.88	0.673976	

Table 3. Possible Combinations of x_{ij} and y_j and Their Corresponding u_2 Values

x_{21}	x_{22}	y_1	y_2	u_2	
0.72	0.72	0.82	0.92	0.720000	MAX.
0.72	0.72	0.82	0.88	0.720000	MAX.
0.72	0.72	0.78	0.92	0.720000	MAX.
0.72	0.72	0.78	0.88	0.720000	MAX.
0.72	0.68	0.82	0.92	0.698851	
0.72	0.68	0.82	0.88	0.699294	
0.72	0.68	0.78	0.92	0.698353	
0.72	0.68	0.78	0.88	0.698795	
0.68	0.72	0.82	0.92	0.701149	
0.68	0.72	0.82	0.88	0.700706	
0.68	0.72	0.78	0.92	0.701647	
0.68	0.72	0.78	0.88	0.701205	
0.68	0.68	0.82	0.92	0.680000	MIN.
0.68	0.68	0.82	0.88	0.680000	MIN.
0.68	0.68	0.78	0.92	0.680000	MIN.
0.68	0.68	0.78	0.88	0.680000	MIN.

Table 4. $\alpha - cut$ Values While $\alpha_0 = 0.00$

$\hat{x}_{12}$	$\hat{x}_{11}$	$\hat{x}_{21}$	$\hat{x}_{22}$	$\hat{y}_1$	$\hat{y}_2$
0.70	0.50	0.60	0.60	0.70	0.80
0.90	0.70	0.80	0.80	0.90	1.00

Table 5. $\alpha - cut$ Values While $\alpha_0 = 0.50$

$\hat{x}_{12}$	$\hat{x}_{11}$	$\hat{x}_{21}$	$\hat{x}_{22}$	$\hat{y}_1$	$\hat{y}_2$
0.75	0.55	0.65	0.65	0.75	0.85
0.85	0.65	0.75	0.75	0.85	0.95

Table 6. $\alpha - cut$ Values While $\alpha_0 = 1.00$

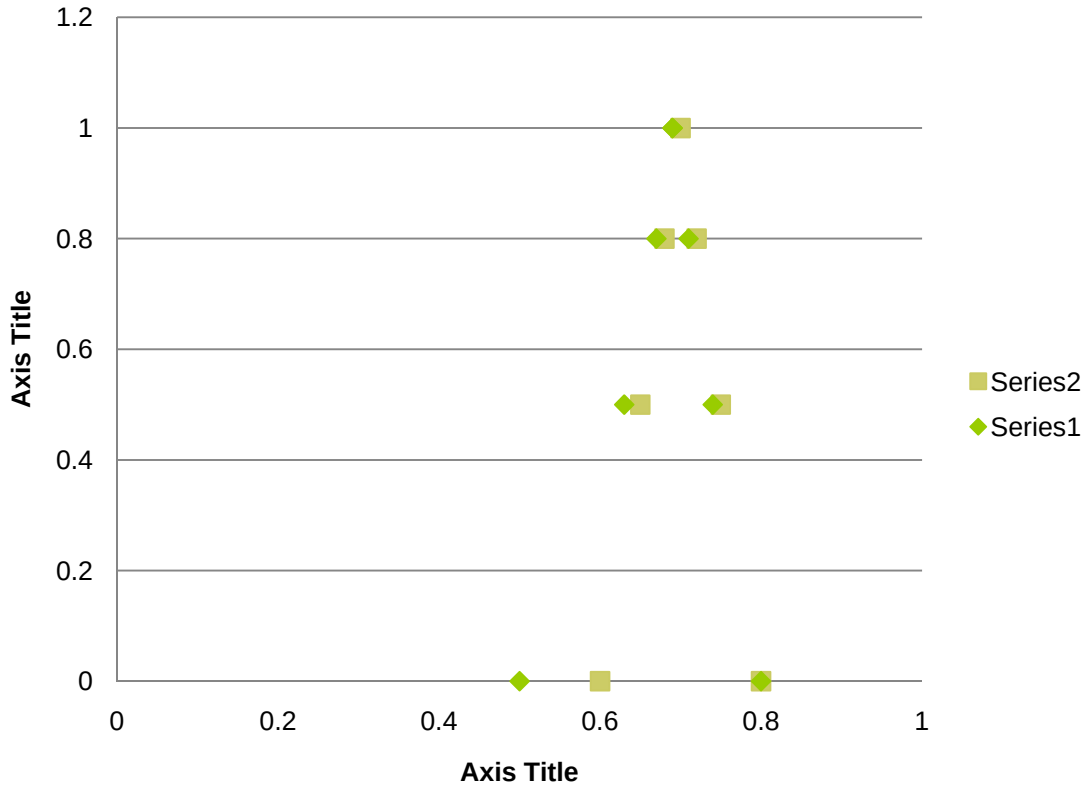
$\hat{x}_{12}$	$\hat{x}_{11}$	$\hat{x}_{21}$	$\hat{x}_{22}$	$\hat{y}_1$	$\hat{y}_2$
0.80	0.60	0.70	0.70	0.80	0.90
0.80	0.60	0.70	0.70	0.80	0.90

Table 7. The Utility Values of u_1

$\mu_{u1}(u_1 = \alpha_0)$	0	0.50	0.80	1.0 $u_1 = \alpha_0)$
u_{1max}	0.805882	0.744444	0.716471	0.694118
u_{1min}	0.582353	0.638235	0.671765	0.694118

Table 8. The Utility Values of u_2

$\mu_{u1}(u_1 = \alpha_0)$	0	0.50	0.80	1.0 $(u_1 = \alpha_0)$
$u2_{max}$	0.80	0.75	0.72	0.70
$u2_{min}$	0.60	0.65	0.68	0.70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Series2
Series1

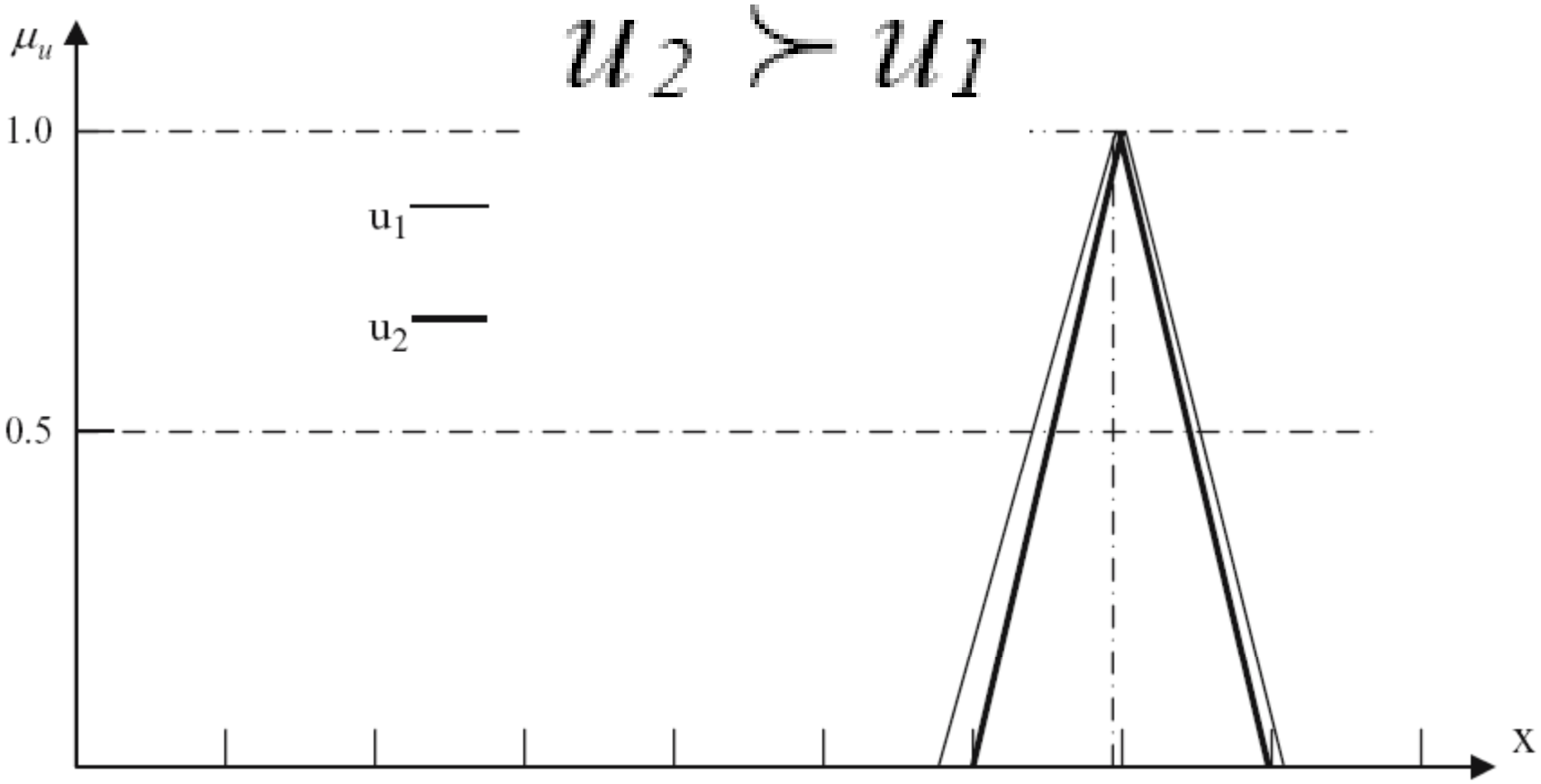

Figure 3. The alternatives' fuzzy utilities using Baas and Kwakernaak's approach (1977)

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Multiplicative weighting method

Multiplicative weighting method

$$\tilde{V}(A_i) = \left(\prod_{j=1}^{n} x_{lij}^{w_{li}}, \prod_{j=1}^{n} x_{mij}^{w_{mi}}, \prod_{j=1}^{n} x_{uij}^{w_{ui}} \right)$$

Criteria	Criteria Weights	FMS-1	FMS-2
engineering effort (X_1)	(0.15, 0.18, 0.24)	(3, 4, 6)	(5, 6, 7)
flexibility (X_2)	(0.32, 0.38, 0.46)	(6, 8, 10)	(4, 4, 5)
net present worth (X_3)	(0.30, 0.32, 0.38)	(28, 32, 44)	(38, 45, 52)
integration ability (X_4)	(0.10, 0.12, 0.18)	(6, 8, 9)	(3, 4, 5)

$$\tilde{V}\left(FMS_1\right) = \left(\prod_{j=1}^{4} x_{l1j}^{w_{l1}}, \prod_{j=1}^{4} x_{m1j}^{w_{m1}}, \prod_{j=1}^{4} x_{u1j}^{w_{u1}} \right)$$

$$= \begin{pmatrix} 3^{0.15} \times 6^{0.32} \times 28^{0.30} \times 6^{0.10}, & 4^{0.18} \times 8^{0.38} \times 32^{0.32} \times 8^{0.12}, \\ 6^{0.24} \times 10^{0.46} \times 44^{0.38} \times 9^{0.18} & \end{pmatrix}$$

$$= \left(6.8005,\ 11.0043\ ,\ 27.7352\right)$$

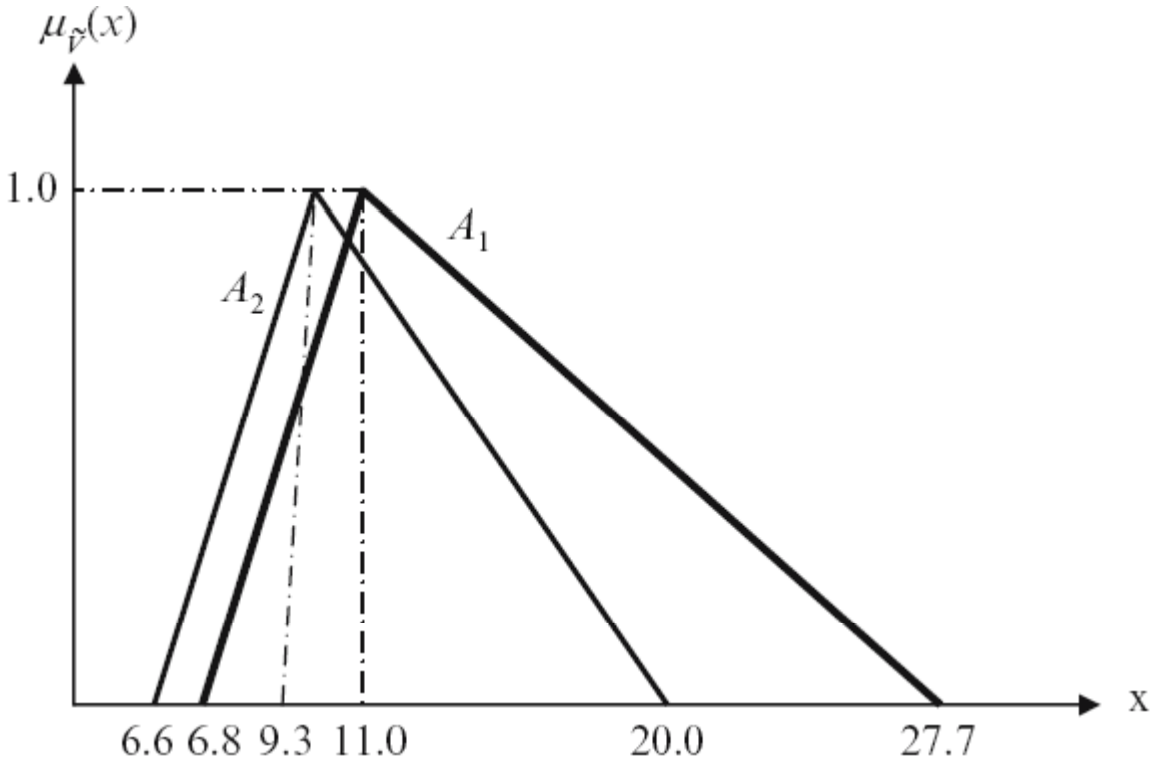
Criteria	Criteria Weights	FMS-1	FMS-2
engineering effort (X_1)	(0.15, 0.18, 0.24)	(3, 4, 6)	(5, 6, 7)
flexibility (X_2)	(0.32, 0.38, 0.46)	(6, 8, 10)	(4, 4, 5)
net present worth (X_3)	(0.30, 0.32, 0.38)	(28, 32, 44)	(38, 45, 52)
integration ability (X_4)	(0.10, 0.12, 0.18)	(6, 8, 9)	(3, 4, 5)

$$\tilde{V}\left(FMS_2\right)=\left(\prod_{j=1}^{4} x_{l2j}^{w_{l2}},\ \prod_{j=1}^{4} x_{m2j}^{w_{m2}},\ \prod_{j=1}^{4} x_{u2j}^{w_{u2}}\right)$$

$$=\begin{pmatrix} 5^{0.15}\times 4^{0.32}\times 38^{0.30}\times 3^{0.10},\ 6^{0.18}\times 4^{0.38}\times 45^{0.32}\times 4^{0.12}, \\ 7^{0.24}\times 5^{0.46}\times 52^{0.38}\times 5^{0.18} \end{pmatrix}$$

$$=\left(6.5940,\ 9.3352,\ 20.0560\right)$$

Criteria	Criteria Weights	FMS-1	FMS-2
engineering effort (X_1)	(0.15, 0.18, 0.24)	(3, 4, 6)	(5, 6, 7)
flexibility (X_2)	(0.32, 0.38, 0.46)	(6, 8, 10)	(4, 4, 5)
net present worth (X_3)	(0.30, 0.32, 0.38)	(28, 32, 44)	(38, 45, 52)
integration ability (X_4)	(0.10, 0.12, 0.18)	(6, 8, 9)	(3, 4, 5)


$\mu_{\tilde{V}}(x)$
1.0
A_1
A_2
x
6.6 6.8 9.3 11.0
20.0
27.7