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# Operations Management

Forecasting

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## Outline

- GLOBAL COMPANY PROFILE: TUPPERWARE CORPORATION
- WHAT IS FORECASTING?
  - Forecasting Time Horizons
  - The Influence of Product Life Cycle
- TYPES OF FORECASTS
- THE STRATEGIC IMPORTANCE OF FORECASTING
  - Human Resources
  - Capacity
  - Supply-Chain Management
- SEVEN STEPS IN THE FORECASTING SYSTEM

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## Outline – Continued

- FORECASTING APPROACHES
  - Overview of Qualitative Methods
  - Overview of Quantitative Methods
- TIME-SERIES FORECASTING
  - Decomposition of Time Series
  - Naïve Approach
  - Moving Averages
  - Exponential Smoothing
  - Exponential Smoothing with Trend Adjustment
  - Trend Projections
  - Seasonal Variations in Data
  - Cyclic Variations in Data

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## Outline – Continued

- **ASSOCIATIVE FORECASTING METHODS: REGRESSION AND CORRELATION ANALYSIS**
  - **Using Regression Analysis to Forecast**
  - **Standard Error of the Estimate**
  - **Correlation Coefficients for Regression Lines**
  - **Multiple-Regression Analysis**
- **MONITORING AND CONTROLLING FORECASTS**
  - **Adaptive Smoothing**
  - **Focus Forecasting**
- **FORECASTING IN THE SERVICE SECTOR**

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## Forecasting at Tupperware

- Each of 50 profit centers around the world is responsible for computerized monthly, quarterly, and 12-month sales projections
- These projections are aggregated by region, then globally, at Tupperware's World Headquarters
- Tupperware uses all techniques discussed in text

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## Three Key Factors for Tupperware

- The number of registered "consultants" or sales representatives
- The percentage of currently "active" dealers (this number changes each week and month)
- Sales per active dealer, on a weekly basis

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## Tupperware – Forecast by Consensus

- Although inputs come from sales, marketing, finance, and production, final forecasts are the consensus of all participating managers.
- The final step is Tupperware's version of the "jury of executive opinion"

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## What is Forecasting?

- Process of predicting a future event
- Underlying basis of all business decisions
  - Production
  - Inventory
  - Personnel
  - Facilities


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## Types of Forecasts by Time Horizon

- Short-range forecast
  - Up to 1 year; usually less than 3 months
  - Job scheduling, worker assignments
- Medium-range forecast
  - 3 months to 3 years
  - Sales & production planning, budgeting
- Long-range forecast
  - $3^{+}$ years
  - New product planning, facility location

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## Short-term vs. Longer-term Forecasting

- *Medium/long range* forecasts deal with more comprehensive issues and support management decisions regarding planning and products, plants and processes.
- *Short-term* forecasting usually employs different methodologies than longer-term forecasting
- *Short-term* forecasts tend to be more accurate than longer-term forecasts.





## Influence of Product Life Cycle

### Introduction, Growth, Maturity, Decline

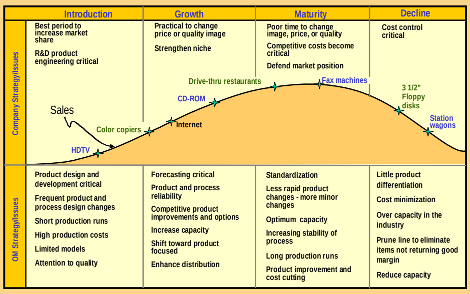
- Stages of introduction and growth require longer forecasts than maturity and decline
- Forecasts useful in projecting
  - staffing levels,
  - inventory levels, and
  - factory capacity

  as product passes through life cycle stages





## Strategy and Issues During a Product's Life



## Types of Forecasts

- Economic forecasts
  - Address business cycle, e.g., inflation rate, money supply etc.
- Technological forecasts
  - Predict rate of technological progress
  - Predict acceptance of *new* product
- Demand forecasts
  - Predict sales of *existing* product





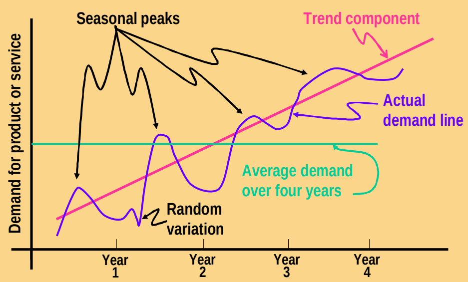
## Seven Steps in Forecasting

- Determine the use of the forecast
- Select the items to be forecasted
- Determine the time horizon of the forecast
- Select the forecasting model(s)
- Gather the data
- Make the forecast
- Validate and implement results





## Product Demand Charted over 4 Years with Trend and Seasonality



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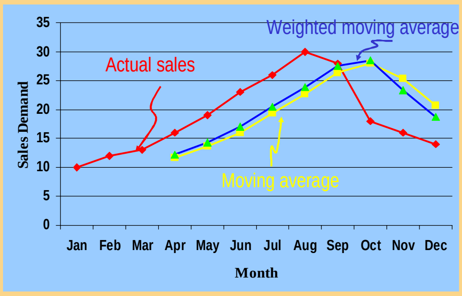
## Actual Demand, Moving Average, Weighted Moving Average

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## Realities of Forecasting

- Forecasts are seldom perfect
- Most forecasting methods assume that there is some underlying stability in the system
- Both product family and aggregated product forecasts are more accurate than individual product forecasts

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## Forecasting Approaches

**Qualitative Methods**

- Used when situation is vague & little data exist
  - New products
  - New technology
- Involves intuition, experience
  - e.g., forecasting sales on Internet

**Quantitative Methods**

- Used when situation is 'stable' & historical data exist
  - Existing products
  - Current technology
- Involves mathematical techniques
  - e.g., forecasting sales of color televisions

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## Overview of Qualitative Methods

- Jury of executive opinion
  - Pool opinions of high-level executives, sometimes augment by statistical models
- Delphi method
  - Panel of experts, queried iteratively
- Sales force composite
  - Estimates from individual salespersons are reviewed for reasonableness, then aggregated
- Consumer Market Survey
  - Ask the customer





## Jury of Executive Opinion

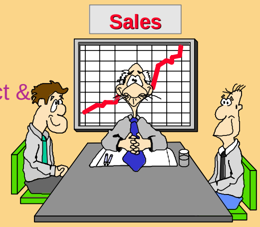
- Involves small group of high-level managers
  - Group estimates demand by working together
- Combines managers' experience with statistical models
- Relatively quick
- 'Group-think' disadvantage





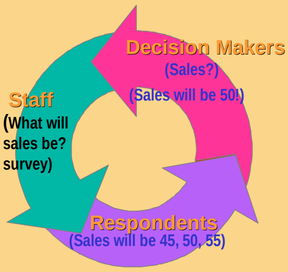
## Sales Force Composite

- Each salesperson projects his or her sales
- Combined at district & national levels
- Sales reps know customers' wants
- Tends to be overly optimistic



## Delphi Method

- Iterative group process
- 3 types of people
  - Decision makers
  - Staff
  - Respondents
- Reduces 'group-think'








## Consumer Market Survey

- Ask customers about purchasing plans
- What consumers say, and what they actually do are often different
- Sometimes difficult to answer






## Overview of Quantitative Approaches

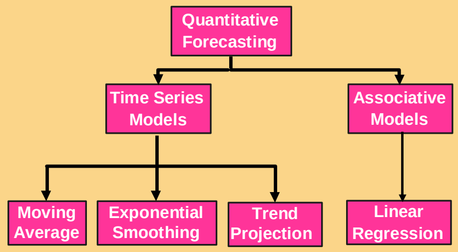
- Naïve approach
- Moving averages
- Exponential smoothing
- Trend projection

} Time-series Models

- Linear regression

} Associative models

## Quantitative Forecasting Methods
(Non-Naive)








## What is a Time Series?

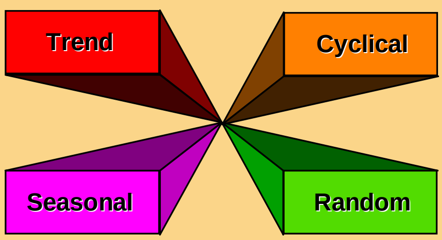
- Set of evenly spaced numerical data
  - Obtained by observing response variable at regular time periods
- Forecast based only on past values
  - Assumes that factors influencing past and present will continue influence in future
- Example

| Year: | 1998 | 1999 | 2000 | 2001 | 2002 |
|---|---|---|---|---|---|
| Sales: | 78.7 | 63.5 | 89.7 | 93.2 | 92.1 |



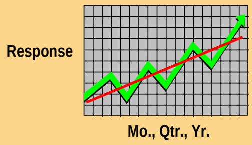


## Time Series Components



# Trend Component

- Persistent, overall upward or downward pattern
- Due to population, technology etc.
- Several years duration

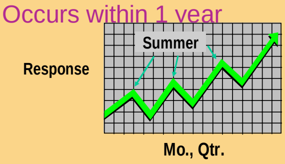







# Seasonal Component

- Regular pattern of up & down fluctuations
- Due to weather, customs etc.
- Occurs within 1 year





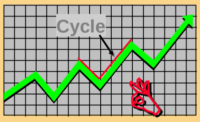



# Common Seasonal Patterns

| Period of Pattern | "Season" Length | Number of "Seasons" in Pattern |
|---|---|---|
| Week | Day | 7 |
| Month | Week | 4 – 4 ½ |
| Month | Day | 28 – 31 |
| Year | Quarter | 4 |
| Year | Month | 12 |
| Year | Week | 52 |

## Cyclical Component

- Repeating up & down movements
- Due to interactions of factors influencing economy
- Usually 2-10 years duration

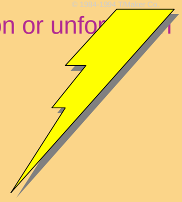







## Random Component

- Erratic, unsystematic, ‘residual’ fluctuations
- Due to random variation or unfor… events
  - Union strike
  - Tornado
- Short duration & nonrepeating







## General Time Series Models

- Any observed value in a time series is the product (or sum) of time series components
- Multiplicative model
  - $Y_i = T_i \cdot S_i \cdot C_i \cdot R_i$ (if quarterly or mo. data)
- Additive model
  - $Y_i = T_i + S_i + C_i + R_i$ (if quarterly or mo. data)

## Naive Approach

- Assumes demand in *next* period is the same as demand in *most recent* period
  - e.g., If May sales were 48, then June sales will be 48
- Sometimes cost effective & efficient





## Moving Average Method

- MA is a series of arithmetic means
- Used if little or no trend
- Used often for smoothing
  - Provides overall impression of data over time
- Equation

$$MA = \frac{\sum \text{Demand in Previous } n \text{ Periods}}{n}$$





## Moving Average Example

You're manager of a museum store that sells historical replicas. You want to forecast sales (000) for *2003* using 3-period moving average.

| | |
|---|---|
| 1998 | 4 |
| 1999 | 6 |
| 2000 | 5 |
| 2001 | 3 |
| 2002 | 7 |

## Moving Average Solution

| Time | Response $Y_i$ | Moving Total (n=3) | Moving Average (n=3) |
|---|---|---|---|
| 1998 | 4 | NA | NA |
| 1999 | 6 | NA | NA |
| 2000 | 5 | NA | NA |
| 2001 | 3 | 4+6+5=15 | 15/3 = 5 |
| 2002 | 7 | | |
| 2003 | NA | | |





## Moving Average Solution

| Time | Response $Y_i$ | Moving Total (n=3) | Moving Average (n=3) |
|---|---|---|---|
| 1998 | 4 | NA | NA |
| 1999 | 6 | NA | NA |
| 2000 | 5 | NA | NA |
| 2001 | 3 | 4+6+5=15 | 15/3 = 5 |
| 2002 | 7 | 6+5+3=14 | 14/3=4 2/3 |
| 2003 | NA | | |

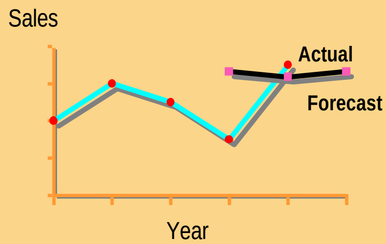




## Moving Average Solution

| Time | Response $Y_i$ | Moving Total (n=3) | Moving Average (n=3) |
|---|---|---|---|
| 1998 | 4 | NA | NA |
| 1999 | 6 | NA | NA |
| 2000 | 5 | NA | NA |
| 2001 | 3 | 4+6+5=15 | 15/3=5.0 |
| 2002 | 7 | 6+5+3=14 | 14/3=4.7 |
| 2003 | NA | 5+3+7=15 | 15/3=5.0 |

# Moving Average Graph








# Weighted Moving Average Method

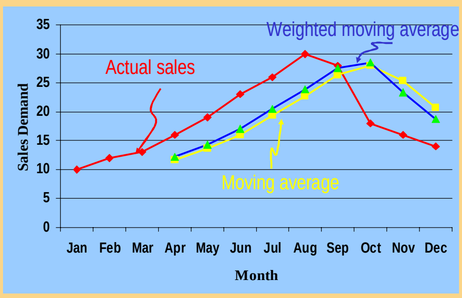
- Used when trend is present
  - Older data usually less important
- Weights based on intuition
  - Often lay between 0 & 1, & sum to 1.0
- Equation

$$WMA = \frac{\Sigma(\text{Weight for period } n)\ (\text{Demand in period } n)}{\Sigma \text{Weights}}$$



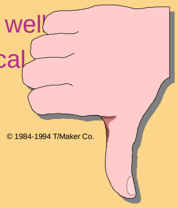


# Actual Demand, Moving Average, Weighted Moving Average



## Disadvantages of Moving Average Methods

- Increasing *n* makes forecast less sensitive to changes
- Do not forecast trend well
- Require much historical data








## Exponential Smoothing Method

- Form of weighted moving average
  - Weights decline exponentially
  - Most recent data weighted most
- Requires smoothing constant ($\alpha$)
  - Ranges from 0 to 1
  - Subjectively chosen
- Involves little record keeping of past data





## Exponential Smoothing Equations

- $F_t = \alpha A_{t-1} + \alpha(1-\alpha)A_{t-2} + \alpha(1-\alpha)^2 \cdot A_{t-3} + \alpha(1-\alpha)^3 A_{t-4} + \ldots + \alpha(1-\alpha)^{t-1} \cdot A_0$
  - $F_t$ = Forecast value
  - $A_t$ = Actual value
  - $\alpha$ = Smoothing constant
- $F_t = F_{t-1} + \alpha(A_{t-1} - F_{t-1})$
  - Use for computing forecast

## Exponential Smoothing Example

During the past 8 quarters, the Port of Baltimore has unloaded large quantities of grain. ($\alpha$ = .10). The first quarter forecast was 175.

| Quarter | Actual |
|---|---|
| 1 | 180 |
| 2 | 168 |
| 3 | 159 |
| 4 | 175 |
| 5 | 190 |
| 6 | 205 |
| 7 | 180 |
| 8 | 182 |
| 9 | ? |

Find the forecast for the 9th quarter.





## Exponential Smoothing Solution

$$F_t = F_{t-1} + 0.1(A_{t-1} - F_{t-1})$$

| Quarter | Actual | Forecast, $F_t$ ($\alpha$ = .10) |
|---|---|---|
| 1 | 180 | 175.00 (Given) |
| 2 | 168 | 175.00 + |
| 3 | 159 | |
| 4 | 175 | |
| 5 | 190 | |
| 6 | 205 | |





## Exponential Smoothing Solution

$$F_t = F_{t-1} + 0.1(A_{t-1} - F_{t-1})$$

| Quarter | Actual | Forecast, $F_t$ ($\alpha$ = .10) |
|---|---|---|
| 1 | 180 | 175.00 (Given) |
| 2 | 168 | 175.00 + .10( |
| 3 | 159 | |
| 4 | 175 | |
| 5 | 190 | |
| 6 | 205 | |

## Exponential Smoothing Solution

$$F_t = F_{t-1} + 0.1(A_{t-1} - F_{t-1})$$

| Quarter | Actual | Forecast, $F_t$ ($\alpha$ = .10) |
|---|---|---|
| 1 | 180 | 175.00 (Given) |
| 2 | 168 | 175.00 + .10(180 - |
| 3 | 159 | |
| 4 | 175 | |
| 5 | 190 | |
| 6 | 205 | |





## Exponential Smoothing Solution

$$F_t = F_{t-1} + 0.1(A_{t-1} - F_{t-1})$$

| Quarter | Actual | Forecast, $F_t$ ($\alpha$ = .10) |
|---|---|---|
| 1 | 180 | 175.00 (Given) |
| 2 | 168 | 175.00 + .10(180 - 175.00) |
| 3 | 159 | |
| 4 | 175 | |
| 5 | 190 | |
| 6 | 205 | |





## Exponential Smoothing Solution

$$F_t = F_{t-1} + 0.1(A_{t-1} - F_{t-1})$$

| Quarter | Actual | Forecast, $F_t$ ($\alpha$ = .10) |
|---|---|---|
| 1 | 180 | 175.00 (Given) |
| 2 | 168 | 175.00 + .10(180 - 175.00) = 175.50 |
| 3 | 159 | |
| 4 | 175 | |
| 5 | 190 | |
| 6 | 205 | |

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## Exponential Smoothing Solution

$F_t = F_{t-1} + 0.1(A_{t-1} - F_{t-1})$

| Quarter | Actual | Forecast, $F_t$ ($\alpha$ = .10) |
|---|---|---|
| 1 | 180 | 175.00 (Given) |
| 2 | 168 | 175.00 + .10(180 - 175.00) = 175.50 |
| 3 | 159 | 175.50 + .10(168 - 175.50) = 174.75 |
| 4 | 175 | |
| 5 | 190 | |
| 6 | 205 | |

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## Exponential Smoothing Solution

$F_t = F_{t-1} + 0.1(A_{t-1} - F_{t-1})$

| Quarter | Actual | Forecast, $F_t$ ($\alpha$ = .10) |
|---|---|---|
| 1995 | 180 | 175.00 (Given) |
| 1996 | 168 | 175.00 + .10(180 - 175.00) = 175.50 |
| 1997 | 159 | 175.50 + .10(168 - 175.50) = 174.75 |
| 1998 | 175 | 174.75 + .10(159 - 174.75)= 173.18 |
| 1999 | 190 | |
| 2000 | 205 | |

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## Exponential Smoothing Solution

$F_t = F_{t-1} + 0.1(A_{t-1} - F_{t-1})$

| Quarter | Actual | Forecast, $F_t$ ($\alpha$ = .10) |
|---|---|---|
| 1 | 180 | 175.00 (Given) |
| 2 | 168 | 175.00 + .10(180 - 175.00) = 175.50 |
| 3 | 159 | 175.50 + .10(168 - 175.50) = 174.75 |
| 4 | 175 | 174.75 + .10(159 - 174.75) = 173.18 |
| 5 | 190 | 173.18 + .10(175 - 173.18) = 173.36 |
| 6 | 205 | |

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## Exponential Smoothing Solution

$F_t = F_{t-1} + 0.1(A_{t-1} - F_{t-1})$

| Quarter | Actual | Forecast, $F_t$ ($\alpha$ = .10) |
|---|---|---|
| 1 | 180 | 175.00 (Given) |
| 2 | 168 | 175.00 + .10(180 - 175.00) = 175.50 |
| 3 | 159 | 175.50 + .10(168 - 175.50) = 174.75 |
| 4 | 175 | 174.75 + .10(159 - 174.75) = 173.18 |
| 5 | 190 | 173.18 + .10(175 - 173.18) = 173.36 |
| 6 | 205 | 173.36 + .10(190 - 173.36) = 175.02 |





## Exponential Smoothing Solution

$F_t = F_{t-1} + 0.1(A_{t-1} - F_{t-1})$

| Time | Actual | Forecast, $F_t$ ($\alpha$ = .10) |
|---|---|---|
| 4 | 175 | 174.75 + .10(159 - 174.75) = 173.18 |
| 5 | 190 | 173.18 + .10(175 - 173.18) = 173.36 |
| 6 | 205 | 173.36 + .10(190 - 173.36) = 175.02 |
| 7 | 180 | 175.02 + .10(205 - 175.02) = 178.02 |
| 8 | | |
| 9 | | |





## Exponential Smoothing Solution

$F_t = F_{t-1} + 0.1(A_{t-1} - F_{t-1})$

| Time | Actual | Forecast, $F_t$ ($\alpha$ = .10) |
|---|---|---|
| 4 | 175 | 174.75 + .10(159 - 174.75) = 173.18 |
| 5 | 190 | 173.18 + .10(175 - 173.18) = 173.36 |
| 6 | 205 | 173.36 + .10(190 - 173.36) = 175.02 |
| 7 | 180 | 175.02 + .10(205 - 175.02) = 178.02 |
| 8 | 182 | 178.02 + .10(180 - 178.02) = 178.22 |
| 9 | ? | 178.22 + .10(182 - 178.22) = 178.58 |

## Forecast Effects of Smoothing Constant α

$$F_t = \alpha A_{t-1} + \alpha(1-\alpha)A_{t-2} + \alpha(1-\alpha)^2 A_{t-3} + \ldots$$

| α= | Weights | | |
|---|---|---|---|
| | Prior Period | 2 periods ago | 3 periods ago |
| | α | α(1 - α) | α(1 - α)² |
| α= 0.10 | 10% | | |
| α= 0.90 | | | |



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## Forecast Effects of Smoothing Constant α

$$F_t = \alpha A_{t-1} + \alpha(1-\alpha) A_{t-2} + \alpha(1-\alpha)^2 A_{t-3} + \ldots$$

| α= | Weights | | |
|---|---|---|---|
| | Prior Period | 2 periods ago | 3 periods ago |
| | α | α(1 - α) | α(1 - α)² |
| α= 0.10 | 10% | 9% | |
| α= 0.90 | | | |



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## Forecast Effects of Smoothing Constant α

$$F_t = \alpha A_{t-1} + \alpha(1-\alpha)A_{t-2} + \alpha(1-\alpha)^2 A_{t-3} + \ldots$$

| α= | Weights | | |
|---|---|---|---|
| | Prior Period | 2 periods ago | 3 periods ago |
| | α | α(1 - α) | α(1 - α)² |
| α= 0.10 | 10% | 9% | 8.1% |
| α= 0.90 | | | |

# Forecast Effects of Smoothing Constant α

$$F_t = \alpha A_{t-1} + \alpha(1-\alpha)A_{t-2} + \alpha(1-\alpha)^2 A_{t-3} + \ldots$$

| α= | Weights | | |
|---|---|---|---|
| | Prior Period | 2 periods ago | 3 periods ago |
| | α | α(1 - α) | α(1 - α)² |
| α= 0.10 | 10% | 9% | 8.1% |
| α= 0.90 | 90% | | |





# Forecast Effects of Smoothing Constant α

$$F_t = \alpha A_{t-1} + \alpha(1-\alpha)A_{t-2} + \alpha(1-\alpha)^2 A_{t-3} + \ldots$$

| α= | Weights | | |
|---|---|---|---|
| | Prior Period | 2 periods ago | 3 periods ago |
| | α | α(1 - α) | α(1 - α)² |
| α= 0.10 | 10% | 9% | 8.1% |
| α= 0.90 | 90% | 9% | |



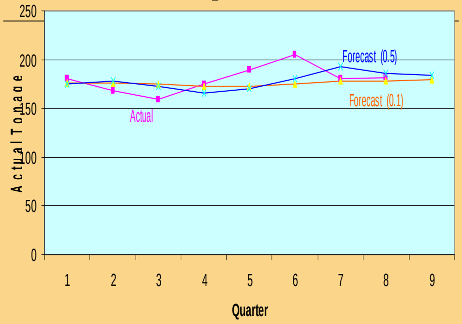


# Forecast Effects of Smoothing Constant α

$$F_t = \alpha A_{t-1} + \alpha(1-\alpha)A_{t-2} + \alpha(1-\alpha)^2 A_{t-3} + \ldots$$

| α= | Weights | | |
|---|---|---|---|
| | Prior Period | 2 periods ago | 3 periods ago |
| | α | α(1 - α) | α(1 - α)² |
| α= 0.10 | 10% | 9% | 8.1% |
| α= 0.90 | 90% | 9% | 0.9% |

# Impact of $\alpha$








# Choosing $\alpha$

Seek to minimize the Mean Absolute Deviation (MAD)

If: Forecast error = demand - forecast

Then:

$$MAD = \frac{\sum |\text{forecast errors}|}{n}$$





# Exponential Smoothing with Trend Adjustment

**Forecast including trend ($FIT_t$)**

**= exponentially smoothed forecast ($F_t$)**

**+ exponentially smoothed trend ($T_t$)**

## Exponential Smoothing with Trend Adjustment – continued

$F_t$ = Last period's forecast
+ $\alpha$(Last period's actual – Last period's forecast)

or

$F_t = F_{t-1} + \alpha (A_{t-1} - F_{t-1})$

$T_t$ = $\beta$(Forecast this period - Forecast last period)
+ (1- $\beta$)(Trend estimate last period

or

$T_t = \beta(F_t - F_{t-1}) + (1- \beta)T_{t-1}$





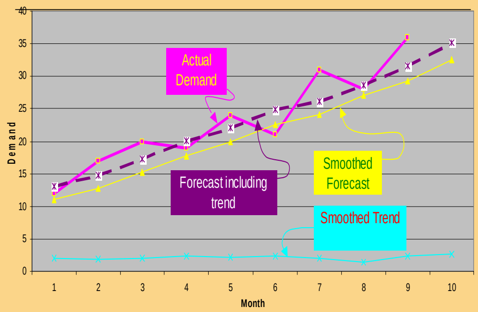
## Exponential Smoothing with Trend Adjustment – continued

- $F_t$ = exponentially smoothed forecast of the data series in period *t*
- $T_t$ = exponentially smoothed trend in period *t*
- $A_t$ = actual demand in period *t*
- $\alpha$ = smoothing constant for the average
- $\beta$ = smoothing constant for the trend





## Comparing Actual and Forecasts

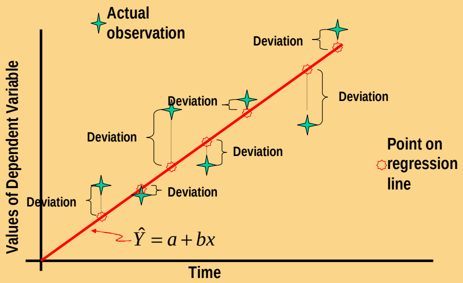


# Regression





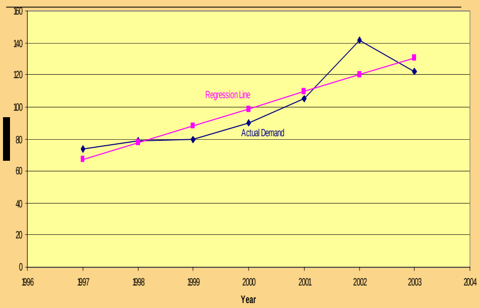
# Least Squares








# Actual and the Least Squares Line



## Linear Trend Projection

- Used for forecasting linear trend line
- Assumes relationship between response variable, *Y*, and time, *X*, is a linear function

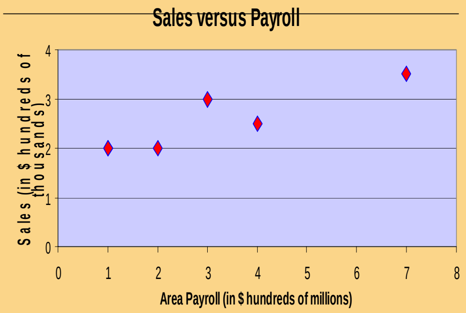
$$Y_i = a + bX_i$$

- Estimated by least squares method
  - Minimizes sum of squared errors





## Scatter Diagram








## Least Squares Equations

**Equation:** $\hat{Y}_i = a + bx_i$

**Slope:** $b = \dfrac{\sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}}{\sum_{i=1}^{n} x_i^2 - n\bar{x}^2}$

**Y-Intercept:** $a = \bar{y} - b\bar{x}$

## Computation Table

| $X_i$ | $Y_i$ | $X_i^2$ | $Y_i^2$ | $X_iY_i$ |
|---|---|---|---|---|
| $X_1$ | $Y_1$ | $X_1^2$ | $Y_1^2$ | $X_1Y_1$ |
| $X_2$ | $Y_2$ | $X_2^2$ | $Y_2^2$ | $X_2Y_2$ |
| : | : | : | : | : |
| $X_n$ | $Y_n$ | $X_n^2$ | $Y_n^2$ | $X_nY_n$ |
| $\Sigma X_i$ | $\Sigma Y_i$ | $\Sigma X_i^2$ | $\Sigma Y_i^2$ | $\Sigma X_iY_i$ |

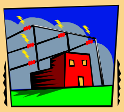




## Using a Trend Line

| Year | Demand |
|---|---|
| 1997 | 74 |
| 1998 | 79 |
| 1999 | 80 |
| 2000 | 90 |
| 2001 | 105 |
| 2002 | 142 |
| 2003 | 122 |

The demand for electrical power at N.Y.Edison over the years 1997 – 2003 is given at the left. Find the overall trend.







## Finding a Trend Line

| Year | Time Period | Power Demand | $x^2$ | xy |
|---|---|---|---|---|
| 1997 | 1 | 74 | 1 | 74 |
| 1998 | 2 | 79 | 4 | 158 |
| 1999 | 3 | 80 | 9 | 240 |
| 2000 | 4 | 90 | 16 | 360 |
| 2001 | 5 | 105 | 25 | 525 |
| 200 | 6 | 142 | 36 | 852 |

# The Trend Line Equation

$$\bar{x} = \frac{\Sigma x}{n} = \frac{28}{7} = 4 \qquad \bar{y} = \frac{\Sigma y}{n} = \frac{692}{7} = 98.86$$

$$b = \frac{\Sigma xy - n\bar{x}\bar{y}}{\Sigma x^2 - n\bar{x}^2} = \frac{3{,}063 - (7)(4)(98.86)}{140 - (7)(4)^2} = \frac{295}{28} = 10.54$$

$$a = \bar{y} - b\bar{x} = 98.86 - 10.54(4) = 56.70$$

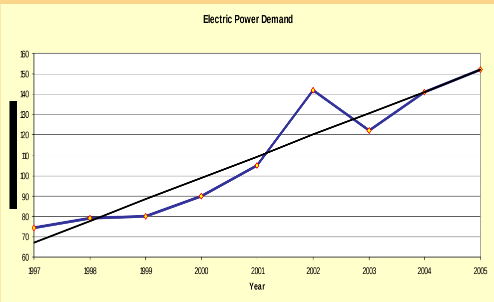
$$\text{Demand in 2004} = 56.70 + 10.54(8) = 141.02 \text{ megawatts}$$

$$\text{Demand in 2005} = 56.70 + 10.54(9) = 151.56 \text{ megawatts}$$





# Actual and Trend Forecast








# Monthly Sales of Laptop Computers

| | Sales Demand | | | Average Demand | | |
|---|---|---|---|---|---|---|
| Month | 2000 | 2001 | 2002 | 2000-2002 | Monthly | Seasonal Index |
| Jan | 80 | 85 | 105 | 90 | 94 | 0.957 |
| Feb | 70 | 85 | 85 | 80 | 94 | 0.851 |
| Mar | 80 | 93 | 82 | 85 | 94 | 0.904 |
| Apr | 90 | 95 | 115 | 100 | 94 | 1.064 |
| May | 113 | 125 | 131 | 123 | 94 | 1.309 |
| Jun | 110 | 115 | 120 | 115 | 94 | 1.223 |
| Jul | 100 | 102 | 113 | 105 | 94 | 1.117 |
| Aug | 88 | 102 | 110 | 100 | 94 | 1.064 |
| Sept | 85 | 90 | 95 | 90 | 94 | 0.957 |
| Oct | 77 | 78 | 85 | 80 | 94 | 0.851 |
| Nov | 75 | 72 | 83 | 80 | 94 | 0.851 |
| Dec | 82 | 78 | 80 | 80 | 94 | 0.851 |

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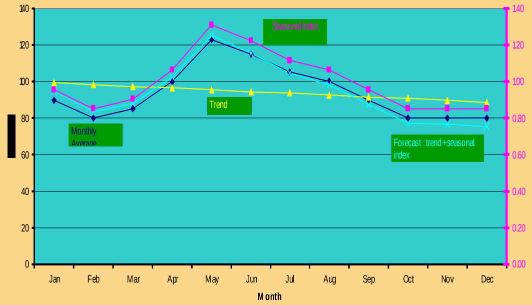
## Demand for IBM Laptops

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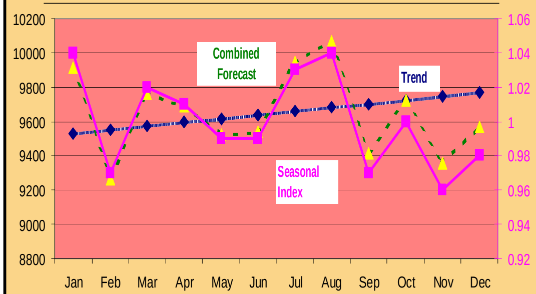
## San Diego Hospital – Inpatient Days

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## Multiplicative Seasonal Model

- Find average historical demand for each "season" by summing the demand for that season in each year, and dividing by the number of years for which you have data.
- Compute the average demand over all seasons by dividing the total average annual demand by the number of seasons.
- Compute a seasonal index by dividing that season's historical demand (from step 1) by the average demand over all seasons.
- Estimate next year's total demand
- Divide this estimate of total demand by the number of seasons, then multiply it by the seasonal index for that season. This provides the seasonal forecast.

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## Linear Regression Model

- Shows linear relationship between dependent & explanatory variables
  - Example: Sales & advertising (*not* time)

$$\hat{Y}_i = a + bX_i$$

Y-intercept: $a$

Slope: $b$

Dependent (response) variable: $\hat{Y}_i$

Independent (explanatory) variable: $X_i$





## Linear Regression Equations

**Equation:** $\hat{Y}_i = a + bx_i$

**Slope:** $$b = \frac{\sum_{i=1}^{n} x_i y_i - n\overline{x}\,\overline{y}}{\sum_{i=1}^{n} x_i^2 - n\overline{x}^2}$$

**Y-Intercept:** $a = \overline{y} - b\overline{x}$





## Computation Table

| $X_i$ | $Y_i$ | $X_i^2$ | $Y_i^2$ | $X_iY_i$ |
|---|---|---|---|---|
| $X_1$ | $Y_1$ | $X_1^2$ | $Y_1^2$ | $X_1Y_1$ |
| $X_2$ | $Y_2$ | $X_2^2$ | $Y_2^2$ | $X_2Y_2$ |
| : | : | : | : | : |
| $X_n$ | $Y_n$ | $X_n^2$ | $Y_n^2$ | $X_nY_n$ |
| $\Sigma X_i$ | $\Sigma Y_i$ | $\Sigma X_i^2$ | $\Sigma Y_i^2$ | $\Sigma X_iY_i$ |

## Interpretation of Coefficients

- Slope (*b*)
  - Estimated *Y* changes by *b* for each 1 unit increase in *X*
    - If $b = 2$, then sales (*Y*) is expected to increase by 2 for each 1 unit increase in advertising (*X*)
- Y-intercept (*a*)
  - Average value of *Y* when $X = 0$
    - If $a = 4$, then average sales (*Y*) is expected to be 4 when advertising (*X*) is 0





## Random Error Variation

- Variation of actual *Y* from predicted *Y*
- Measured by standard error of estimate
  - Sample standard deviation of errors
  - Denoted $S_{Y,X}$
- Affects several factors
  - Parameter significance
  - Prediction accuracy





## Least Squares Assumptions

- Relationship is assumed to be linear. Plot the data first - if curve appears to be present, use curvilinear analysis.
- Relationship is assumed to hold only within or slightly outside data range. Do not attempt to predict time periods far beyond the range of the data base.
- Deviations around least squares line are assumed to be random.

## Standard Error of the Estimate

$$S_{y,x} = \sqrt{\frac{\sum_{i=1}^{n}(y_i - y_c)^2}{n-2}}$$

$$= \sqrt{\frac{\sum_{i=1}^{n} y_i^2 - a\sum_{i=1}^{n} y_i - b\sum_{i=1}^{n} x_i y_i}{n-2}}$$





## Correlation

- Answers: '*how strong* is the linear relationship between the variables?'
- Coefficient of correlation Sample correlation coefficient denoted *r*
  - Values range from -1 to +1
  - Measures degree of association
- Used mainly for understanding

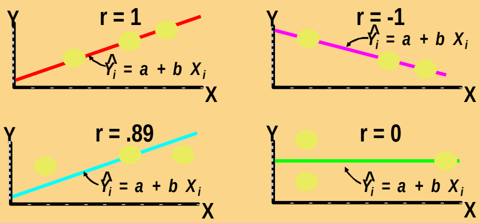




## Sample Coefficient of Correlation

$$r = \frac{n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{\sqrt{\left[n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2\right]\left[n\sum_{i=1}^{n} y_i^2 - \left(\sum_{i=1}^{n} y_i\right)^2\right]}}$$

## Coefficient of Correlation and Regression Model




$r^2$ = square of correlation coefficient (r), is the percent of the variation in *y* that is explained by the regression equation





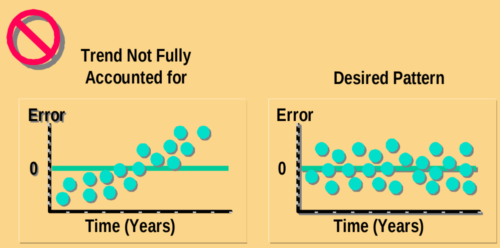
## Guidelines for Selecting Forecasting Model

- You want to achieve:
  - No pattern or direction in forecast error
    - Error = $(Y_i - \hat{Y}_i)$ = (Actual – Forecast)
    - Seen in plots of errors over time
  - Smallest forecast error
    - Mean square error (MSE)
    - Mean absolute deviation (MAD)





## Pattern of Forecast Error



## Forecast Error Equations

- Mean Square Error (MSE)

$$MSE = \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{n} = \frac{\sum \text{forecast errors}^2}{n}$$

- Mean Absolute Deviation (MAD)

$$MAD = \frac{\sum_{i=1}^{n} | y_i - \hat{y}_i |}{n} = \frac{\sum | \text{forecast errors} |}{n}$$

- Mean Absolute Percent Error (MAPE)

$$MAPE = 100 \frac{\sum_{i=1}^{n} \frac{| \text{actual}_i - \text{forecast}_i |}{\text{actual}_i}}{n}$$





## Selecting Forecasting Model Example

You're a marketing analyst for Hasbro Toys. You've forecast sales with a linear model & exponential smoothing. Which model do you use?

| Year | Actual Sales | Linear Model Forecast | Exponential Smoothing Forecast (.9) |
|---|---|---|---|
| 1998 | 1 | 0.6 | 1.0 |
| 1999 | 1 | 1.3 | 1.0 |
| 2000 | 2 | 2.0 | 1.9 |
| 2001 | 2 | 2.7 | 2.0 |
| 2002 | 4 | 3.4 | 3.8 |





## Linear Model Evaluation

| Year | $Y_i$ | $\hat{Y}_i$ | Error | $Error^2$ | \|Error\| | \|Error\| / Actual |
|---|---|---|---|---|---|---|
| 1998 | 1 | 0.6 | 0.4 | 0.16 | 0.4 | 0.40 |
| 1999 | 1 | 1.3 | -0.3 | 0.09 | 0.3 | 0.30 |
| 2000 | 2 | 2.0 | 0.0 | 0.00 | 0.0 | 0.00 |
| 2001 | 2 | 2.7 | -0.7 | 0.49 | 0.7 | 0.35 |
| 2002 | 4 | 3.4 | 0.6 | 0.36 | 0.6 | 0.15 |
| Total | | | 0.0 | 1.10 | 2.0 | 1.20 |

MSE $= \Sigma$ Error$^2$ / $n$ = 1.10 / 5 = 0.220

MAD $= \Sigma$ |Error| / $n$ = 2.0 / 5 = 0.400

MAPE = 100 $\Sigma$|absolute percent errors|/$n$= 1.20/5 = 0.240

## Exponential Smoothing Model Evaluation

| Year | $Y_i$ | $\hat{Y}_i$ | Error | $Error^2$ | \|Error\| | \|Error\| / Actual |
|---|---|---|---|---|---|---|
| 1998 | 1 | 1.0 | 0.0 | 0.00 | 0.0 | 0.00 |
| 1999 | 1 | 1.0 | 0.0 | 0.00 | 0.0 | 0.00 |
| 2000 | 2 | 1.9 | 0.1 | 0.01 | 0.1 | 0.05 |
| 2001 | 2 | 2.0 | 0.0 | 0.00 | 0.0 | 0.00 |
| 2002 | 4 | 3.8 | 0.2 | 0.04 | 0.2 | 0.05 |
| Total | | | 0.3 | 0.05 | 0.3 | 0.10 |

MSE $= \Sigma\ Error^2 / n = 0.05 / 5 = 0.01$

MAD $= \Sigma\ |Error| / n = 0.3 / 5 = 0.06$

MAPE = 100 $\Sigma$ |Absolute percent errors|/$n$ = 0.10/5 = 0.02





## Exponential Smoothing Model Evaluation

**Linear Model:**

MSE $= \Sigma\ Error^2 / n = 1.10 / 5 = .220$

MAD $= \Sigma\ |Error| / n = 2.0 / 5 = .400$

MAPE = 100 $\Sigma$|absolute percent errors|/$n$= 1.20/5 = 0.240

**Exponential Smoothing Model:**

MSE $= \Sigma\ Error^2 / n = 0.05 / 5 = 0.01$

MAD $= \Sigma\ |Error| / n = 0.3 / 5 = 0.06$

MAPE = 100 $\Sigma$ |Absolute percent errors|/$n$ = 0.10/5 = 0.02





## Tracking Signal

- Measures how well the forecast is predicting actual values
- Ratio of running sum of forecast errors (RSFE) to mean absolute deviation (MAD)
  - Good tracking signal has low values
- Should be within upper and lower control limits

## Tracking Signal Equation

$$TS = \frac{RSFE}{MAD}$$

$$= \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)}{MAD}$$

$$= \frac{\sum |forecast\ \ error|}{MAD}$$





## Tracking Signal Computation

| Mo | Fcst | Act | Error | RSFE | Abs Error | Cum \|Error\| | MAD | TS |
|---|---|---|---|---|---|---|---|---|
| 1 | 100 | 90 | | | | | | |
| 2 | 100 | 95 | | | | | | |
| 3 | 100 | 115 | | | | | | |
| 4 | 100 | 100 | | | | | | |
| 5 | 100 | 125 | | | | | | |
| 6 | 100 | 140 | | | | | | |





## Tracking Signal Computation

| Mo | Forc | Act | Error | RSFE | Abs Error | Cum \|Error\| | MAD | TS |
|---|---|---|---|---|---|---|---|---|
| 1 | 100 | 90 | -10 | | | | | |
| 2 | 100 | 95 | | | | | | |
| 3 | 100 | 115 | | | | | | |
| 4 | 100 | 100 | | | | | | |
| 5 | 100 | 125 | | | | | | |
| 6 | 100 | 140 | | | | | | |

Error = Actual - Forecast
= 90 - 100 = -10

## Tracking Signal Computation

| Mo | Forc | Act | Error | RSFE | Abs Error | Cum \|Error\| | MAD | TS |
|---|---|---|---|---|---|---|---|---|
| 1 | 100 | 90 | -10 | -10 | | | | |
| 2 | 100 | 95 | | | | | | |
| 3 | 100 | 115 | | | | | | |
| 4 | 100 | 100 | | | | | | |
| 5 | 100 | 125 | | | | | | |
| 6 | 100 | 140 | | | | | | |

RSFE = Σ Errors
= NA + (-10) = -10





## Tracking Signal Computation

| Mo | Forc | Act | Error | RSFE | Abs Error | Cum \|Error\| | MAD | TS |
|---|---|---|---|---|---|---|---|---|
| 1 | 100 | 90 | -10 | -10 | 10 | | | |
| 2 | 100 | 95 | | | | | | |
| 3 | 100 | 115 | | | | | | |
| 4 | 100 | 100 | | | | | | |
| 5 | 100 | 125 | | | | | | |
| 6 | 100 | 140 | | | | | | |

Abs Error = |Error|
= |-10| = 10





## Tracking Signal Computation

| Mo | Forc | Act | Error | RSFE | Abs Error | Cum \|Error\| | MAD | TS |
|---|---|---|---|---|---|---|---|---|
| 1 | 100 | 90 | -10 | -10 | 10 | 10 | | |
| 2 | 100 | 95 | | | | | | |
| 3 | 100 | 115 | | | | | | |
| 4 | 100 | 100 | | | | | | |
| 5 | 100 | 125 | | | | | | |
| 6 | 100 | 140 | | | | | | |

Cum |Error| = Σ |Errors|
= NA + 10 = 10

## Tracking Signal Computation

| Mo | Forc | Act | Error | RSFE | Abs Error | Cum \|Error\| | MAD | TS |
|---|---|---|---|---|---|---|---|---|
| 1 | 100 | 90 | -10 | -10 | 10 | 10 | 10.0 | |
| 2 | 100 | 95 | | | | | | |
| 3 | 100 | 115 | | | | | | |
| 4 | 100 | 100 | | | | | | |
| 5 | 100 | 125 | | | | | | |
| 6 | 100 | 140 | | | | | | |

MAD = Σ |Errors|/n
= 10/1 = 10





## Tracking Signal Computation

| Mo | Forc | Act | Error | RSFE | Abs Error | Cum \|Error\| | MAD | TS |
|---|---|---|---|---|---|---|---|---|
| 1 | 100 | 90 | -10 | -10 | 10 | 10 | 10.0 | -1 |
| 2 | 100 | 95 | | | | | | |
| 3 | 100 | 115 | | | | | | |
| 4 | 100 | 100 | | | | | | |
| 5 | 100 | 125 | | | | | | |
| 6 | 100 | 140 | | | | | | |

TS = RSFE/MAD
= -10/10 = -1





## Tracking Signal Computation

| Mo | Forc | Act | Error | RSFE | Abs Error | Cum \|Error\| | MAD | TS |
|---|---|---|---|---|---|---|---|---|
| 1 | 100 | 90 | -10 | -10 | 10 | 10 | 10.0 | -1 |
| 2 | 100 | 95 | -5 | | | | | |
| 3 | 100 | 115 | | | | | | |
| 4 | 100 | 100 | | | | | | |
| 5 | 100 | 125 | | | | | | |
| 6 | 100 | 140 | | | | | | |

Error = Actual - Forecast
= 95 - 100 = -5

## Tracking Signal Computation

| Mo | Forc | Act | Error | RSFE | Abs Error | Cum \|Error\| | MAD | TS |
|---|---|---|---|---|---|---|---|---|
| 1 | 100 | 90 | -10 | -10 | 10 | 10 | 10.0 | -1 |
| 2 | 100 | 95 | -5 | -15 | | | | |
| 3 | 100 | 115 | | | | | | |
| 4 | 100 | 100 | | | | | | |
| 5 | 100 | 125 | | | | | | |
| 6 | 100 | 140 | | | | | | |

RSFE = Σ Errors
= (-10) + (-5) = -15





## Tracking Signal Computation

| Mo | Forc | Act | Error | RSFE | Abs Error | Cum \|Error\| | MAD | TS |
|---|---|---|---|---|---|---|---|---|
| 1 | 100 | 90 | -10 | -10 | 10 | 10 | 10.0 | -1 |
| 2 | 100 | 95 | -5 | -15 | 5 | | | |
| 3 | 100 | 115 | | | | | | |
| 4 | 100 | 100 | | | | | | |
| 5 | 100 | 125 | | | | | | |
| 6 | 100 | 140 | | | | | | |

Abs Error = |Error|
= |-5| = 5





## Tracking Signal Computation

| Mo | Forc | Act | Error | RSFE | Abs Error | Cum \|Error\| | MAD | TS |
|---|---|---|---|---|---|---|---|---|
| 1 | 100 | 90 | -10 | -10 | 10 | 10 | 10.0 | -1 |
| 2 | 100 | 95 | -5 | -15 | 5 | 15 | | |
| 3 | 100 | 115 | | | | | | |
| 4 | 100 | 100 | | | | | | |
| 5 | 100 | 125 | | | | | | |
| 6 | 100 | 140 | | | | | | |

Cum Error = Σ |Errors|
= 10 + 5 = 15

## Tracking Signal Computation

| Mo | Forc | Act | Error | RSFE | Abs Error | Cum \|Error\| | MAD | TS |
|---|---|---|---|---|---|---|---|---|
| 1 | 100 | 90 | -10 | -10 | 10 | 10 | 10.0 | -1 |
| 2 | 100 | 95 | -5 | -15 | 5 | 15 | 7.5 | |
| 3 | 100 | 115 | | | | | | |
| 4 | 100 | 100 | | | | | | |
| 5 | 100 | 125 | | | | | | |
| 6 | 100 | 140 | | | | | | |

MAD = Σ |Errors|/n = 15/2 = 7.5





## Tracking Signal Computation

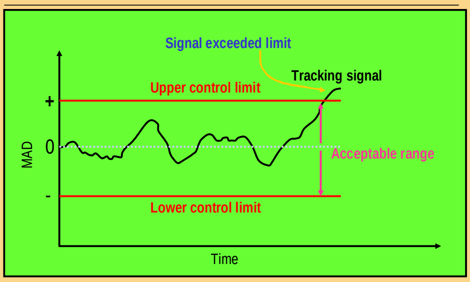
| Mo | Forc | Act | Error | RSFE | Abs Error | Cum \|Error\| | MAD | TS |
|---|---|---|---|---|---|---|---|---|
| 1 | 100 | 90 | -10 | -10 | 10 | 10 | 10.0 | -1 |
| 2 | 100 | 95 | -5 | -15 | 5 | 15 | 7.5 | -2 |
| 3 | 100 | 115 | | | | | | |
| 4 | 100 | 100 | | | | | | |
| 5 | 100 | 125 | | | | | | |
| 6 | 100 | 140 | | | | | | |

TS = RSFE/MAD = -15/7.5 = -2

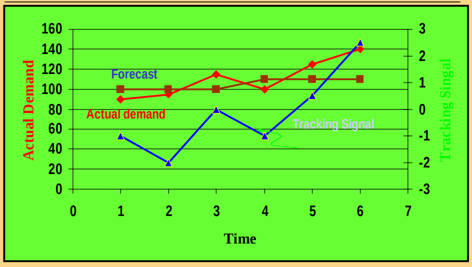




## Plot of a Tracking Signal



## Tracking Signals








## Forecasting in the Service Sector

- Presents unusual challenges
  - special need for short term records
  - needs differ greatly as function of industry and product
  - issues of holidays and calendar
  - unusual events





## Forecast of Sales by Hour for Fast Food Restaurant