Chapter 9

THE DESIGN OF PRODUCTION-DISTRIBUTION NETWORKS: A MATHEMATICAL PROGRAMMING APPROACH

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Abstract This text proposes a mathematical programming approach to design international production-distribution networks for make-to-stock products with convergent manufacturing processes. Various formulations of the elements of production-distribution network design models are discussed. The emphasis is put on modeling issues encountered in practice which have a significant impact on the quality of the logistics network designed. The elements discussed include the choice of an objective function, the definition of the planning horizon, the manufacturing process and product structures, the logistics network structure, demand and service requirements, facility layouts and capacity options, product flows and inventory modeling, as well as financial flows modeling. Major contributions from the literature are reviewed and a number of new formulation elements are introduced. A typical model is presented, and the use of successive mixed-integer programming to solve it with commercial solvers is discussed. A more general version of the model presented and the solution method described were implemented in a commercial supply chain design tool which is now available on the market.

Keywords: Logistics network design, Supply chain engineering, Location-allocation problems, Capacity planning, Technology selection, Mathematical programming.

1. Context

How many production and distribution centers should a company have to satisfy the demand of its targeted markets? Where should they be located and what should their mission be? What supply sources should they use? What technologies should they install for production, storage, shipping and receiving? Which sub-contractors and public warehouses should they do business with? What means of transportation should they choose? All of these questions are related to strategic and tactical logistics network design issues, which are critical for the success of modern manufacturing and distribution companies. This text proposes a mathematical programming approach to analyze several of these logistics network design issues.

The exact nature of the logistics network design problems encountered in practice depends very much on the industrial context in which they occur. For example:

- The design problem to solve for a high volume consumer goods manufacturer is very different than the problem found in a highly customized make-to-order products industry or in a slow moving repair parts distribution context. In a make-to-stock industry, the order-to-delivery time depends on the positioning of finished goods inventories but, in a make-to-order context, it depends on manufacturing lead times and on the depth of penetration of customer orders in the supply chain, i.e., on the positioning of semi-finished product or raw material inventories.

- When manufacturing resource acquisition, deployment and/or allocation decisions are considered, the nature of the production process must also be taken into account. In some industries, manufacturing processes are divergent: several products are made from a common raw material (e.g. pulp and paper industry, meat industry, etc.). In other sectors the manufacturing processes are convergent: several raw-materials and components are assembled into finished products. In some industries, the manufacturing processes may even include feedback loops.

- Networks covering several countries lead to much more complex design problems than single-country networks. Factors such as exchange rates, transfer prices, duties and income taxes must then be taken into account.

The detailed discussion of all these variants is beyond the scope of this paper. In what follows our coverage focuses on the design of international production-distribution networks for make-to-stock products with convergent manufacturing processes.

As can be seen, logistics network design problems, as defined here, integrate several subproblems which have been treated separately in the literature: capital investment planning for the acquisition of new capacity, technology selection, facility location and manufacturing/distribution resource allocation problems. Capacity expansion problems are usually posed as multi-year capital investments problems under uncertainty (Freidenfelds, 1981; Luss, 1982). The financial planning aspects of the problem, such as real options (Trigeorgis, 1996), are predominant in the analysis and the logistics aspects are highly aggregated. Technology selection problems can be seen as an extension of capacity planning where there are several alternative capacity types available (Fine, 1993, Paquet et al., 2004). At the other extreme, resource allocation problems deal with detailed plant loading and inventory placement decisions under the assumption that the plant/warehouse network configuration is fixed (Glover et al., 1979; Cohen and Moon, 1991; Mazzola and Schantz, 1997). They often consider a single year planning horizon divided into several seasons. The literature on basic discrete location models (Francis et al., 1992; Daskin, 1995; Sule, 2001) concentrates on single period, single echelon, geographical deployment problems. A lot of the effort in this field has been devoted to finding efficient solution methods for a set of well defined problems. Some extensions to classical facility location problems are reviewed by Revelle et al. (1996) and by Owen et al. (1998). An abundant literature exists on location, capacity acquisition and technology selection problems. An integrated review of the early work done in these fields is found in Verter and Dincer (1992). Supply chain design models incorporate elements of all the sub-problems discussed previously. Geoffrion and Powers (1995) and Shapiro et al. (1993) discuss the evolution of strategic supply chain design models and Vidal and Goetschalckx (1997) present many of these models. Shapiro (2001) provides an excellent coverage of several supply chain modeling issues.

In this paper, various formulations of the elements of production-distribution network design models are discussed. The emphasis is put on modeling issues encountered in practice which have a significant impact on the quality of the logistics network designed. The elements discussed include the choice of an objective function, the definition of the planning horizon, the manufacturing process and product structures, the logistics network structure, demand and service requirements, facility layouts and capacity options, product flows and inventory modeling, as well as financial flows modeling. Major contributions from the literature are reviewed and a number of new formulation elements are introduced. A typical model is presented, and the use of successive mixed-integer programming to solve it with commercial solvers is discussed. A more general version of the model proposed and the solution method described were implemented in the *Supply Chain Studio*, a commercial supply chain design tool sold by *Modellium.* This tool was used to optimize the production-distribution network of several multinational companies, including Domtar, one of the largest Pulp and Paper Company in North-America.

2. Modeling approach

Performance evaluation

Although most of the logistics network design models presented in the literature adopt a total system cost minimization objective, this does not necessary lead to the creation of a competitive advantage. Low cost is an order winning criteria valued by several customers but it is not the only one (Hill, 1999). Delivery time, quality and flexibility are other valued criteria which are affected by the logistics activities and resources of the firm. In a make-to-stock industry, for example, the order-to-delivery time depends on the positioning of finished goods inventories in the logistics network and it is a criteria as important as cost for the evaluation of network designs. As explained by Porter (1985), it is the additional value given by customers to such an order winning criterion that creates a competitive advantage. Figure 9.1 illustrates the cost accumulation process and the impact of inventory positioning on customer delivery times for a simple multi-echelon (stage) supply chain. As can be seen, costs accumulate as the products pass through the procurement, production and distribution stages, and value is added when the finished products are purchased by customers. The cost of support activities can be interpreted here as all the non-logistic costs incurred by the firm. The response time depends on whether the customers are served from a local or regional warehouse or from a production/distribution center or, more generally, on the distance between a customer and its supply facility. When delivery time is shorter, more revenues are generated through a price premium and/or an increased market share. Total system cost, maximum delivery time and total revenue figures are therefore associated with any logistics network design. In order to evaluate the performance of various designs, their cost and delivery time can be plotted on a graph, as shown in Figure 9.2a). The non-dominated designs are located on an *efficient-frontier*, and any of these designs could constitute a good solution for a firm (Rosenfield, 1985). However, if the impact of delivery time on prices and on demand, and thus on total revenue, is taken into account, as shown in Figure 9.2b), the design maximizing the value added (Total revenue − Total logistics network cost − Cost of support activities) by the logistics network can be identified. Ideally, the objective to pursue should therefore be to find the logistics network design maximizing *net revenues.* In an international context, since different countries have different taxation levels, one should rather seek to maximize after tax global net revenues in a reference currency. Unfortunately, it is not always possible in practice to model the impact of delivery time on price and demand. When this is the case, one should at least sketch the efficient frontier by finding the designs minimizing total system costs for a set of predetermined delivery times. Despite the fact that an abundant literature exists on the impact that quality and flexibility may have on competitiveness, little work has been done to explicitly incorporate them as performance criterion in logistics network design models. By associating different technologies to different quality levels, quality can often be treated in a way similar to delivery times. Some dimensions of flexibility, such as operational flexibility in global networks under exchange rate risk (Kogut and Kulatilaka, 1994; Huchzermeier and Cohen, 1996), have been studied, but more research is needed on the incorporation of the various dimensions of flexibility into network design models. The model presented in what follows seeks to maximize after tax net revenues, taking the impact of delivery times on revenues into account.

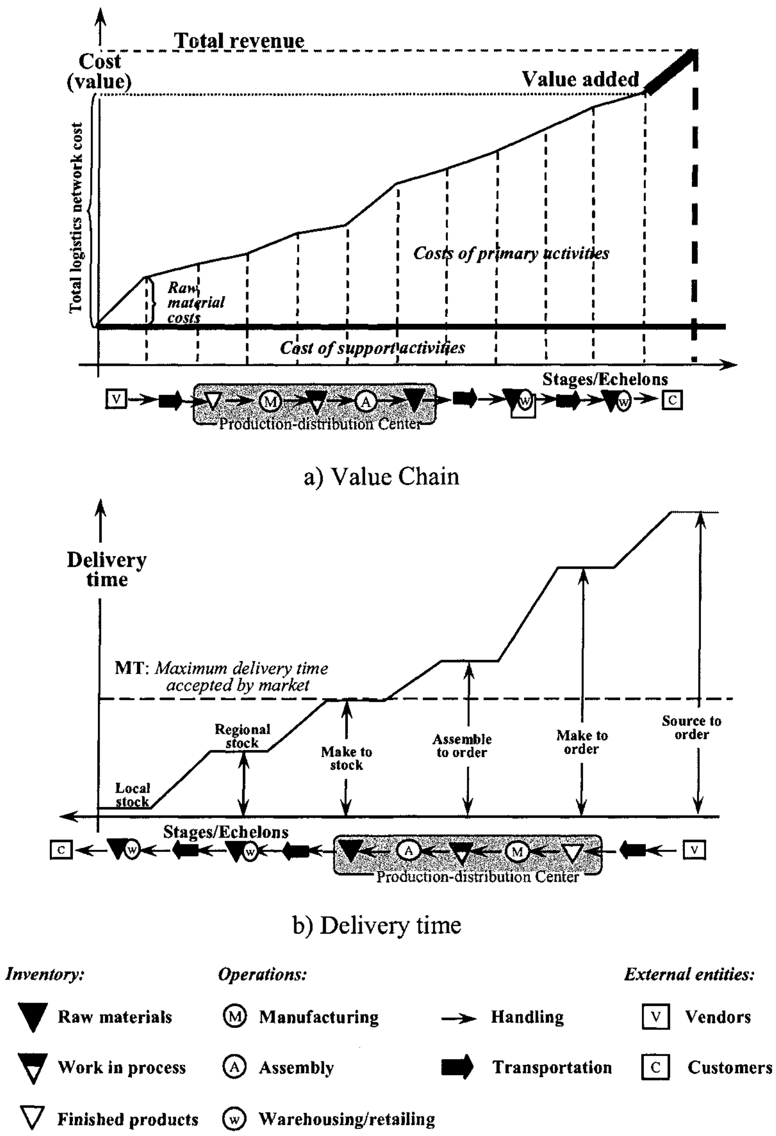

Figure 9.1. Costs, value added and delivery time in the supply chain.

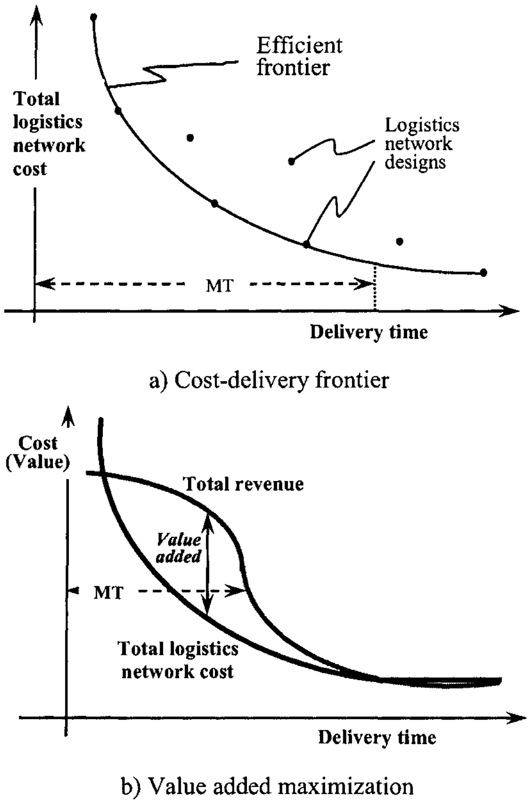

Figure 9.2. Performance evaluation methods.

Planning horizon and uncertainty

In capital intensive industries, capacity expansion decisions may require the explicit consideration of a planning horizon including as much as ten years (Everett et al., 2000, 2001). On the other end, when product supply and/or demand is seasonal, decisions on production and inventory levels for each network location must be made on a quarterly basis or even on a monthly basis. This means that the number of planning periods in logistics network design models could be very large. In addition to the explosion of problem size, using a long planning horizon makes the gathering of meaningful information on the future business environment extremely difficult. Some approach to reduce this complexity must therefore be used in practice.

To clarify this issue, let us first make a distinction between the notions of *season* and *period*. In most design models, 0-1 variables are associated with capacity acquisition and deployment decisions and continuous variables to resource allocation decisions (production and inventory levels, network flows). A *multi-period model* is concerned with the change of state of the network structure (number, location, technology and capacity of facilities) over the long term (typically several one year periods). A *multi-season model* is concerned with the change of mission of the network resources during a planning period (typically months or quarters during a year). Several formulations presented as multi-period models in the literature are in fact single-period multi-season models (Cohen et al., 1989; Arntzen et al., 1995; Dogen and Goetschalckx, 1999). Multi-period models usually concentrate on capacity investment decisions and they limit themselves to single echelon network structures (Shulman, 1991, Everett et al., 2000; Bhutta et al., 2003). Following the pioneering work of Pomper (1976), some authors have also proposed multi-period scenario based stochastic programming models (Eppen et al., 1989; Ahmed et al., 2001; Everett et al., 2001).

Most of the models published in the literature are deterministic single-period mathematical programs (Geoffrion and Graves, 1974; Brown et al., 1987; Cohen and Lee, 1989; Cohen and Moon, 1990; Pirkul and Jayaraman, 1996; Lakhal et al., 2001; Vidal and Goetschalckx, 2001; Cordeau et al., 2002; Paquet et al., 2004). It is understood, however, that since the acquisition and deployment decisions have long-term effects, their analyses must span multiple periods and the model must be either run sequentially over some finite time horizon or, when size permits, expanded to incorporate multiple time periods directly (Cohen and Lee, 1989). Also, the fact that the future is uncertain requires the examination of several scenarios with respect to the firm's strategic options and the evolution of its internal and external environment (Shapiro, 2001) or, when size permits, the transformation of the model into a multi-stage stochastic program with recourse (Birge and Louveaux, 1997). Keeping this in mind, the approach presented in what follows yields deterministic multi-season logistics network design models. The following set is used to denote the planning horizon:

T = Seasons of the planning horizon ($t \in T$).

Modeling process and product structures

In order to arrive at a general production-distribution network design model for a given industrial context, a generic conceptual model of the manufacturing process of the industry must first be elaborated. Such a conceptual model treats products and production stages in an aggregate manner to capture the essence of the manufacturing process, but without concern for operational details (Shapiro, 2001). It can take the form of an activity network or of a bill-of-materials, as illustrated in Figure 9.3 (Lakhal et al., 1999). In these conceptual models, products are grouped into product families and some activities may be an amalgam of several operations. It is common to use process network representations in process manufacturing environments such as petro-chemicals, food, pulp and paper, pharmaceutical, etc. (Brown et al., 1987; Dogan and Goetschalckx, 1999; Philpott and Everett, 2001; Vila et al., 2003). In such contexts, associated with each activity are a number of methods (recipes) that describe how inputs are transformed into outputs using different potential technologies. In discrete parts manufacturing industries, however, a bill-of-materials representation is usually more adequate (Cohen and Moon, 1990; Arntzen et al., 1995; Paquet et al., 2004). This is the approach taken in this paper.

More specifically, the following product structure modeling assumptions are made. Products are classified in families $p \in P$ requiring the same type of production capacity or supplied by the same vendors. Products available only from external suppliers are considered as raw material (RM) and other products can be manufactured in the network plants. The manufactured products (MP) are sub-assemblies (SA) or make-to-stock (MS) finished products. The semifinished products can come partly from external suppliers and partly from the network plants. The aggregated bill-of-materials, illustrated in Figure 9.3b), is an acyclic directed graph. The number associated with the edge $(p'p)$ of the bill-of-materials graph indicates the quantity of the product p' needed to make one product of type p. It is assumed that the vertices of this graph are numbered in topological order, i.e., that for each edge $(p'p)$, we have $p > p'$. A *technology* is defined by the set of products it can manufacture/store, and it is assumed that the bill-of-materials is independent of the technology used. As illustrated in Figure 9.3b), the capacity required to produce one product can be provided by either flexible or dedicated

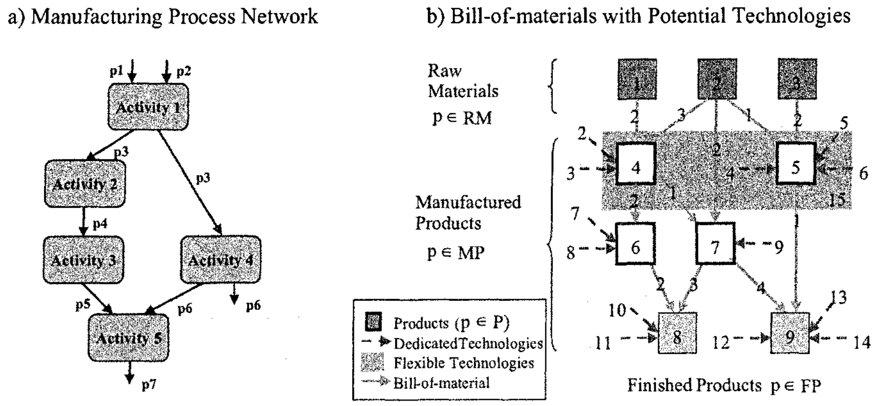

Figure 9.3. Process and product structures.

technologies. Dedicated technologies are associated with only one product family, but flexible technologies can be used to make several product families. Similarly, the capacity needed to stock the finished products can be provided by a set of potential storage technologies. When a facility is used, a technology for the reception and shipping of products must also be implemented. To simplify, it is assumed that this technology can be used for any products and that its capacity can be expressed adequately in terms of the facility outflows.

The notation used to model product structures and technologies is the following:

P = Product families ($p \in P$).

RM = Raw material families ($RM \subset P$).

MP = Manufactured product families, i.e., sub-assemblies and finished products ($MP \subset P$).

SA = Sub-assemblies families ($SA \subset P$).

$g_{pp'}$ = Quantity of product p needed to make one product p'.

KW = Receiving/shipping/handling technologies ($k \in KW$).

KM = Production technologies ($k \in KM$).

KS = Storage technologies ($k \in KS$).

q_{pk} = Technology k capacity consumption rate per unit of product p.

$\overline{w}_p$ = Average weight of family p products in standard weight units.

Network optimization model structure

The structure of logistics networks can be represented by a directed graph. The network nodes correspond to supply sources, to existing facilities, to sites where it would be possible to build or buy a production or distribution center, to the facilities of potential partners (subcontractors, public warehouses, 3PL consolidation centers, etc.) or to demand zones. The network arcs represent the flow of products between the nodes. The specification of the structure of the network and of the mission of its facilities is an important strategic decision. Two approaches to the problem are found in the literature, as illustrated in Figure 9.4. A popular modeling approach has been to assume *a priori* that a multi-echelon structure is required (Geoffrion and Graves, 1974; Cohen and Lee, 1989; Pirkul and Jayaraman, 1996; Vidal and Goetschalckx, 2001). This limits the mission of the facilities to a predetermined role (e.g. intermediate product plant, final product plant, distribution center) and it forbids product flows between facilities on the same echelon. In some contexts, this approach can be far from optimal. In practice, the same facility often has multiple roles: a production-distribution center may produce both intermediate and finished products and serve as a shipping point to some customers; a warehouse close to a supplier may serve as a central warehouse for this supplier's products, but as a local distribution center for other products, etc. For this reason, other authors do not impose any *a priori* echelon structure and expect the optimization model to determine the best structure and mission for the facilities (Arntzen et al., 1995; Paquet et al., 2004).

Two approaches are also used in the literature to model flows in the network. One of them associates decision variables to *paths* in the network (Geoffrion and Graves, 1974; Martel and Vankatadri, 1999). This is particularly appropriate for multi-echelon distribution networks. However, for this approach to work, the product flowing on the path must not change between the first node and the last node, which cannot hold for production facilities. For this reason, models incorporating more than one production echelon either associate decision variables to the *arcs* of the network (Arntzen et al., 1995, Dogan and Goetschalckx, 1999; Vidal and Goetschalckx, 2001; Cordeau, 2002, Paquet et al., 2004), or they

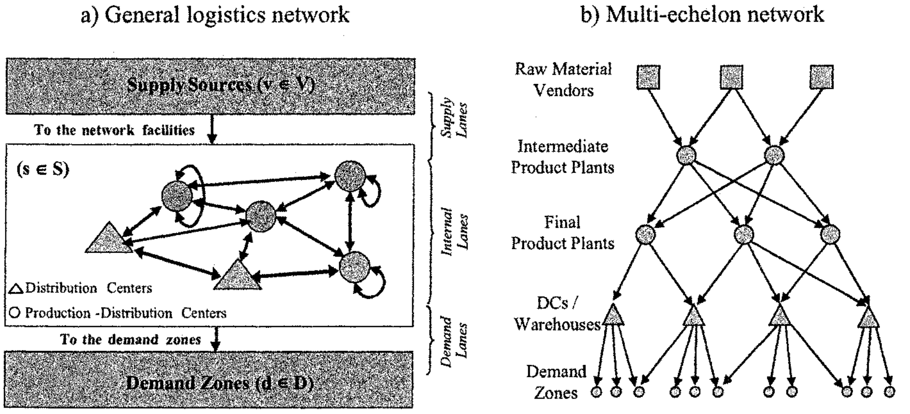

Figure 9.4. Potential logistics network.

use a hybrid approach (Cohen and Moon, 1990). The model presented in this paper is an arc-based formulation for the general logistics network illustrated in Figure 9.4a). Three types of nodes, located in several countries, are present in the network: external vendors ($v \in V$), internal potential facility sites ($s \in S$) and demand zones ($d \in D$). A list of potential internal sites (S) must be identified *a priori* and classified as either production-distribution center sites (Spd) or distribution center sites (Sd). This list usually includes the location of the current facilities, of public warehouses or sub-contractors which could be included in the network, of existing facilities which could be purchased or rented, and of lands where a new facility could be constructed. It is possible also to limit the mission given to potential sites by restricting the set of production (KM_s) and storage (KS_s) technologies which can be implemented in a site, or the set of products (P_s) which can be produced/stored in a site. The network arcs are associated with transportation lanes. Three types of arcs are distinguished: supply arcs, internal arcs and demand arcs. The internal arcs adjacent to a site s are defined by the set of origins of its inbound arcs (S^i_{ps}) and the set of destinations of its outbound arcs (S^o_{ps}). Similar node input and output sets are defined for supply and demand arcs. A continuous decision variable F_{pnst} is associated with the flow of a product p on lane (n, s) in season t. Given that a real logistics network may include several hundred thousand arcs, defining these sets and flow variables in practice is not trivial and it requires the use of an automated arc generation mechanism.

The customer ship-to locations are grouped into demand zones (D). The definition of these demand zones depends on the product-markets (M) of the company and on the geographical dispersion of ship-to points (Ballou, 1994). It is assumed that the company operates national divisions in several countries $o \in O$, and that each of these divisions covers a set of distinct product-markets $m \in M_o$ constituted of several demand zones $d \in D_m$. A market is characterized by a distinct price and service policy. It is assumed that the products shipped to a demand zone can come from more than one distribution center. This is common today because companies tend to operate centralized selling organizations independent of the DC's. Modifying the model however to enforce single DC sourcing is not difficult. Similarly, vendors in close geographic proximity who provide products in the same family can be aggregated into a supply source (V). It is assumed that the seasonal quantity of product which can be supplied by a vendor is bounded. The following sets, indices, parameters and variables are required to define a potential logistics network:

S = Potential network sites ($s \in S$).

O = Countries of the network sites ($o \in O, o(s) =$ country of site s).

S_o = Potential sites in country o.

Sd = Potential distribution center sites ($Sd \subset S$).

Spd = Potential production-distribution center sites ($Spd \subset S$).

Sd_{max} = Upper bound on the number of distribution centers in the network.

Spd_{max} = Upper bound on the number of production-distribution centers in the network.

V_p = Vendors of raw material $p \in RM$ or of manufactured product $p \in MP$.

b_{pvt} = Upper bound on the quantity of raw material p which can be supplied by vendor v in season t.

S^o_{pn} = Set of potential sites (output destinations) which can receive product p from node n.

S^i_{ps} = Set of potential sites (input sources) which can ship product p to site s.

M_o = Potential product-markets in country o ($m \in M = \cup_{o \in O} M_o$).

D_m = Demand zones in product-market m ($d \in D = \cup_{m \in M} D_m$).

D_o = Demand zones in country o ($D_o = \cup_{m \in M_o} D_m$).

$m(d)$ = Product-market of demand zone d.

D^o_{ps} = Set of demand zones (output destinations) which can receive product p from node s.

P_s = Products which can be manufactured/stocked on site s.

P_{ks} = Products which can be manufactured/stocked with technology k on site s.

KM_{ps} = Production technologies which can be used to manufacture product p on site s ($KM_{ps} \subset KM, KM_s = \cup_p KM_{ps}$).

KS_{ps} = Storage technologies which can be used to stock product p on site s ($KS_{ps} \subset KS,\ KS_s = \cup_p KS_{ps}$).

F_{pnst} = Flow of product p between node $n \in V_p \cup S^i_{ps}$ and site s during season t.

The essence of the logistics network design problem boils down to finding an optimal mapping of the product/activity structure onto the potential network structure.

Modeling demand, prices and customer service

Although most of the models available in the literature assume that demand is given and not affected by the logistics network design, this is clearly not realistic. As explained earlier, demand depends on logistics outputs such as delivery times, and the market may be prepared to pay a price premium to obtain these outputs. To take this into account, it is assumed that the company has a choice of marketing policies $i \in I_m$ for each of its product-markets m (Vila et al., 2004). A marketing policy $i \in I_m$ is characterized by the price P_{pdit} the market is prepared to pay for each product $p \in P$ in the demand zones $d \in D_m$ during seasons $t \in T$. It is also characterized by a maximum delivery time and possibly by other value criteria. These value criteria are related to the network design by defining the set of sites in the potential network S^i_{pdi} which could deliver the value characteristics of marketing policy $i \in I_{m(d)}$, for each product p. It is further assumed that the largest demand $\overline{x}_{pdit}$ the company can expect for product p in demand zone d, when marketing policy $i \in I_{m(d)}$ is used, can be estimated, and that the company has minimum market penetration objectives $\underline{x}_{pdt}$ for each of its product-markets.

In this context, the following notation is required to model the demand:

I_m = Marketing policies considered for market m ($i \in I_m$).

S^i_{pdi} = Set of potential sites (input sources) which can ship product p to demand zone d, when marketing policy $i \in I_{m(d)}$ is selected.

P_{pdit} = Amount received for the sales of product p to demand zone d in season t when marketing policy $i \in I_{m(d)}$ is used (in the demand zone country currency).

$\underline{x}_{pdt}$ = Lower bound on the flow of product p to demand zone d in season t imposed by the market penetration objectives of the company.

$\overline{x}_{pdit}$ = Upper bound on the flow of product p to demand zone d in season t imposed by the largest market share the company can expect when marketing policy $i \in I_{m(d)}$ is used.

Y^M_{mi} = Binary variable equal to 1 if marketing policy $i \in I_m$ is used for market m and to 0 otherwise.

F_{psdit} = Flow of product p between site s and demand zone d during season t, when marketing policy $i \in I_{m(d)}$ is selected.

Parallel arcs are defined between the network sites s and the demand zones d to model the flow of products F_{psdit} under the different marketing policies $i \in I_{m(d)}$. Using these flow variables and the marketing policy selection variables Y^M_{mi}, it is seen that the seasonal sale targets

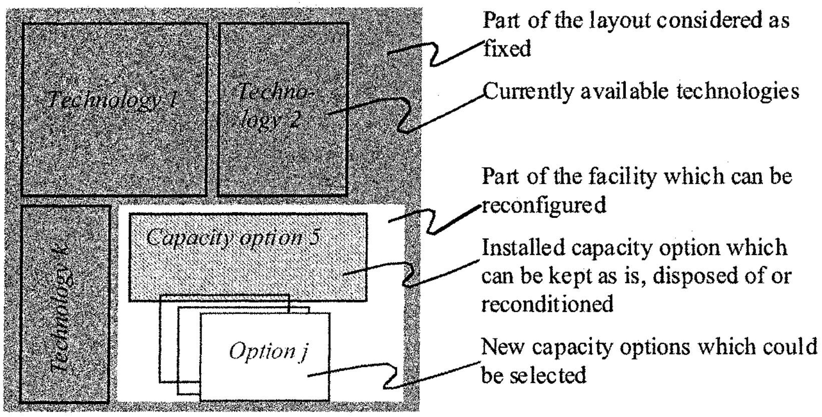

Figure 9.5. Illustration of the Facility Layout Concept.

of the company must respect the following demand and policy selection constraints:

$$\underline{x}_{pdt} Y^M_{m(d)i} \leq \sum_{s \in S^i_{pdi}} F_{psdit} \leq \overline{x}_{pdit} Y^M_{m(d)i} \quad t \in T, p \in P, d \in D, i \in I_{m(d)} \quad (9.1)$$

$$\sum_{i \in I_m} Y^M_{mi} \leq 1 \quad m \in M \quad (9.2)$$

Modeling facility layouts and capacity options

The technical and economic characteristics of the facilities which could be operated on the network sites can be specified with a *facility layout*. The facility layout concept is illustrated schematically in Figure 9.5. A layout $l \in L_s$ for site s is composed of two parts: a fixed part, which cannot be changed and a variable part defining an area which could be reengineered. The technologies implemented in the fixed part are predetermined and they specify the products they can make/stock, the seasonal capacity available b_{lskt}, stated in the units of its technology, and the associated variable costs. The variable part defines an area E_{ls} available for the installation of a set of predetermined *capacity options*. A facility layout may include only a fixed or a variable part. Several layouts can be considered for each site s, including a *status-quo* layout if there is already a facility on the site, and alternative potential layouts corresponding to new construction or reconfiguration opportunities. Numerous capacity options can be available to implement a given technology in the variable part of a layout. An option $j \in J$ can correspond to capacity already in place, to a reconfiguration of an installed equipment to increase its capacity or to the addition of new resources. In this last case, different options can be associated with equipment of different size to reflect economies of scale. Moreover, the simultaneous inclusion of dedicated capacity options and flexible capacity options allow for the modeling of economies of scope. When dealing with a potential equipment replacement/reconfiguration, the options associated with the new potential equipment cannot be selected at the same time as the *status-quo* option, which leads to the definition of mutually exclusive sub-sets of options JR^n_{ls}, $n = 1, ..., N_{ls}$, for some facility layouts. Each option $j \in J$ is characterized by a seasonal capacity, b_{jt}, stated in the units of its technology, by the floor space e_j required to install it, as well as by a fixed cost and a variable cost per product.

The notation required to include layout and option choice decisions in the model is the following:

L_s = Potential facility layouts for site s ($l \in L_s$). By convention, the index $l = 1$ is given to the current layout if there is a facility on site s at the beginning of the horizon.

L_{ks} = Potential facility layouts including fixed technology k capacity for site s ($l \in L_s$).

b_{lskt} = Technology k capacity available for season t in the fixed part of layout l of site s.

E_{ls} = Total area of the variable part of layout l for site s.

Y_{ls} = Binary variable equal to 1 if layout l is used on site s and to 0 otherwise.

Y_{0s} = Binary variable equal to 1 if site s is not used and to 0 otherwise.

J_s = Potential capacity options which can be installed on site s ($j \in J = \cup_{s \in S} J_s$).

J_{ks} = Potential technology k capacity options which can be installed on site s ($J_{ks} \subseteq J_s$).

J_{ls} = Potential capacity options which can be installed on site s when layout l is used ($J_{ls} \subseteq J_s$).

N_{ls} = Number of mutually exclusive option subsets (equipment replacement/reconfiguration) in J_{ls}.

JR_{ls}^n = Mutually exclusive option subsets in J_{ls} ($n = 1, ..., N_{ls}$).

$k(j)$ = Technology of capacity option j.

b_{jt} = Technology $k(j)$ capacity provided by option j for season t.

e_j = Area required to install capacity option j.

Z_j = Binary variable equal to 1 if capacity option j is installed and to 0 otherwise.

Using the layout selection variables Y_{ls}, the following constraints must be included in the model to ensure that at most one layout is selected for each site, and that the total number of facilities used does not exceed the maximum number of distribution and production-distribution centers desired:

$$\textstyle\sum_{l \in L_s} Y_{ls} + Y_{0s} = 1 \qquad s \in S \tag{9.3}$$

$$\sum_{s \in Sd} \sum_{l \in L_s} Y_{ls} \leq Sd_{max} \tag{9.4}$$

$$\sum_{s \in Spd} \sum_{l \in L_s} Y_{ls} \leq Spd_{max} \tag{9.5}$$

Using the capacity option selection variables, Z_j, the following constraints must also be included to ensure that, for a given site, the area required by the selected options does not exceed the area available in the selected layout, and that mutually exclusive options are not selected:

$$\textstyle\sum_{j \in J_{ls}} e_j Z_j - E_{ls} Y_{ls} \leq 0 \quad s \in S, l \in L_s \tag{9.6}$$

$$\textstyle\sum_{j \in JR_{ls}^n} Z_j \leq 1 \qquad s \in S, l \in L_s, n = 1, \ldots, N_{ls} \tag{9.7}$$

Modeling flows and inventories

In addition to deciding the marketing policies, sites, layouts and capacity options to use during the planning horizon, tactical seasonal decisions must be made on the quantity of products to manufacture, the seasonal stocks to accumulate and the flows in the network. This requires the modeling of flows and inventories in the network facilities and the consideration of capacity constraints. Several types of facilities are used in practice and the flow patterns in and between the centers, as well as the nature of the inventory kept, can be quite different from one type of facility to the other. To simplify, it is assumed here that there is a single type of production-distribution center (P-DC) and a single type of distribution center (DC) in the network. The structure of the P-DC's considered is illustrated in Figure 9.6: they include different production technologies and they can manufacture any component or finished products associated with these technologies in the bill-of-materials (Figure 9.3b). When the manufacturing of a product is completed, it is either used to make other products, moved to the facility inventory or shipped to another facility. It is assumed that there is no seasonal inventory of input products and that the plant warehouse contains only products to be shipped directly to the market. All products made for other internal centers are shipped directly to these facilities after production. On the other hand, it is assumed that DC's can receive products from vendors or from any other site, and that they can ship products to the market or to any other site.

The additional notation required to model flows and inventories is the following:

I_{psd} = Sub-set of the demand zone d marketing policies which include site s as a valid supply site for product p $(I_{psd} = \{i | s \in S^i_{pdi}\} \subset I_{m(d)})$.

$\underline{W}_s$ = Lower bound on the seasonal throughput, in standard weight units, required to use distribution center s.

$\overline{X}_{pst}$ = Upper bound on the quantity of family p products which can be manufactured in plant s during season t.

$\underline{X}_{pst}$ = Lower bound on the quantity of family p products which can be manufactured in plant s during season t, when plant s is used.

$\rho^{(0)}_{pst}$ = Number of seasons of product p order cycle and safety stocks kept on average in site s during season$t$ (inverse of the inventory turnover ratio).

β_p = Order cycle and safety stocks (max. level)/(avg. level) ratio for product p.

X_{pkst} = Quantity of product p produced in plant s with technology

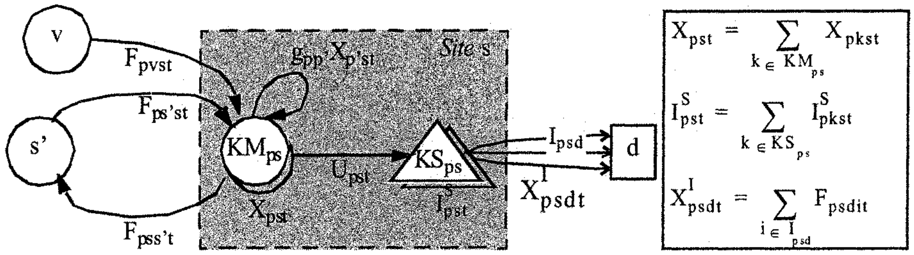

Figure 9.6. Flow of sub-assembly $p \in SA$ in a production-distribution center.

$k \in KM_{ps}$ during season t.

U_{pst} = Quantity of product p transferred to the stock of site s during season t.

I^S_{pkst} = Seasonal inventory of product p stored in site s with technology $k \in KS_{ps}$ at the end of season t.

X^I_{pst} = Throughput of product p in distribution center $s \in S$ for season t.

Any valid network optimization model must ensure that there is equilibrium between the flows of products entering a node, their transformation, stocking and/or consumption in the node and the flows of products exiting the node. The case of a sub-assembly $p \in SA$ manufactured in center $s \in Spd$ is illustrated in Figure 9.6, for a season t. In the part of facility s used by the production technologies (KM_{ps}), the quantity of sub-assembly p manufactured (X_{pst}) must be sufficient to satisfy the needs of the other network sites ($F_{pss't}$), the transfers to the seasonal stock (U_{pst}), and the sub-assembly requirements generated by client products in the bill-of-materials ($g_{pp'}X_{p'st}$, $\forall p' > p$, i.e., by all products p' including subassembly p), taking into account the sub-assemblies coming from other internal sites ($F_{ps'st}$, $\forall s' \neq s$) and from external suppliers (F_{pvst}, $\forall v \in V_p$). In order to have flow equilibrium, the following relations must therefore be satisfied:

$$\sum_{k \in KM_{ps}} X_{pkst} + \sum_{n \in S^i_{ps} \cup V_p} F_{pnst} \geq$$
$$\sum_{s' \in S^o_{ps}} F_{pss't} + \sum_{p'>p} g_{pp'} \sum_{k \in KM_{p's}} X_{p'kst} + U_{pst} \quad t \in T, p \in MP, s \in Spd \quad (9.8)$$

Similarly, in the part of facility s used by the storage technologies (KS_{ps}), additions and withdrawals from the seasonal inventory must be

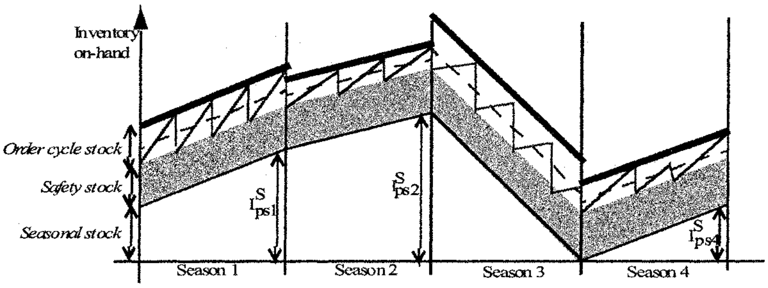

Figure 9.7. Behavior of product p inventory in a distribution center.

accounted for. This yields the following inventory accounting equations:

$$\sum_{k\in KS_{ps}} I^S_{pkst-1} + U_{pst} = X^I_{pst} + \sum_{k\in KS_{ps}} I^S_{pkst} \quad t\in T, p\in MP,$$
$$\left(I^S_{pks0} = I^S_{pks|T|}\right) \quad s\in Spd \qquad (9.9)$$

where

$$X^I_{pst} = \sum_{d\in D^o_{ps}} \sum_{i\in I_{psd}} F_{psdit} \quad t\in T, p\in P, s\in Spd \qquad (9.10)$$

Seasonal stocks are included in the model to allow the smoothing of production over the planning horizon. As illustrated in Figure 9.7, the seasonal stocks at the beginning and the end of the horizon must therefore be the same, i.e., we must have $I^S_{ps0} = I^S_{ps|T|}$, for all p and s.

The quantity X_{pst} of products which can be manufactured in a given P-DC is limited by the layout and the capacity options selected for that center. This imposes the following capacity constraints:

$$\sum_{p\in P_{ks}} q_{pk} X_{pkst} \le \sum_{l\in L_{ks}} b_{lskt} Y_{ls} + \sum_{j\in J_{ks}} b_{jt} Z_j \quad t\in T, s\in Spd,$$
$$k\in KM_s \qquad (9.11)$$

In some contexts, it may also be necessary to bound the quantity of products manufactured in a facility, which can be done with the constraints:

$$\underline{X}_{pst} \sum_{l\in L_s} Y_{ls} \le \sum_{k\in KM_{ps}} X_{pkst} \le \overline{X}_{pst} \sum_{l\in L_s} Y_{ls} \quad t\in T, p\in P_s,$$
$$s\in Spd \qquad (9.12)$$

To simplify the presentation, it is assumed that offset trade and local content rules do not restrict national production. However, the inclusion of constraints to that effect would not present any difficulty (Cohen et al., 1989; Arntzen et al., 1995). For raw materials, the flow equilibrium constraints required for the P-DC's are:

$$\sum_{n \in V_p \cup S^i_{ps}} F_{pnst} \geq \sum_{s' \in S^o_{ps}} F_{pss't} + \sum_{p'>p} g_{pp'} \sum_{k \in KM_{p's}} X_{p'kst} + X^I_{pst} \qquad t \in T, p \in RM, s \in Spd \quad (9.13)$$

For distribution centers, the flow equilibrium constraints and the inventory accounting equations required are the following:

$$\sum_{n \in V_p \cup S^i_{ps}} F_{pnst} \geq X^I_{pst} \quad t \in T, p \in RM, s \in Sd \qquad (9.14)$$

$$\sum_{k \in KS_{ps}} I^S_{pkst-1} + \sum_{n \in S^i_{ps} \cup V_p} F_{pnst} = X^I_{pst} + \sum_{k \in KS_{ps}} I^S_{pkst} \quad t \in T, p \in MP, \quad \left(I^S_{pks0} = I^S_{pks|T|}\right) \quad s \in Sd \qquad (9.15)$$

where

$$X^I_{pst} = \sum_{n \in S^o_{ps}} F_{pnst} + \sum_{d \in D^o_{ps}} \sum_{i \in I_{psd}} F_{psdit} \quad t \in T, p \in P, \quad s \in Sd \qquad (9.16)$$

Also, in most contexts, management does not want to operate small DC's. This leads to the imposition of the following lower bounds on DC throughput:

$$\sum_{p \in P_s} \overline{w}_p X^I_{pst} \geq \underline{W}_s \sum_{l \in L_s} Y_{ls} \quad t \in T, s \in Sd \qquad (9.17)$$

The three types of inventories to take into account in the model are represented in Figure 9.7: seasonal stocks, safety stocks and order cycle stocks. The level of order cycle stocks and of safety stocks depends on the inventory management policies and rules used by the company and on the ordering behavior of customers. Using inventory theory it can be shown (Martel, 2002) that, for a given product supply lead time, the relationship between the seasonal flow of a product in a warehouse and the average level of cycle and safety stocks required to support this

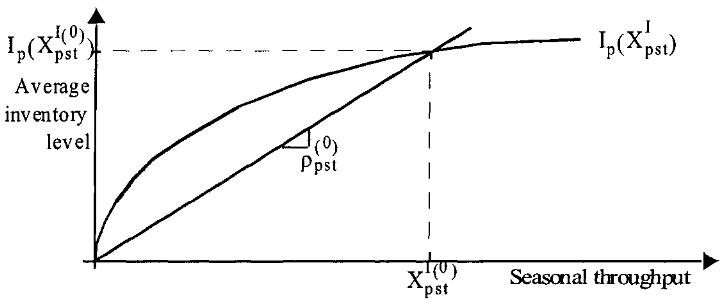

Figure 9.8. Relationship between inventory levels and material flows in a DC.

flow is concave. To simplify things, in what follows, the effect of delivery lead times is assumed to be negligible (see Martel and Vankatadri (1999) for a model incorporating lead times). More specifically, it is assumed that the average inventory level of product p required during season t in warehouse s to support the throughput X^I_{pst} is given by the power function:

$$I_p\left(X^I_{pst}\right) = a_p\left(X^I_{pst}\right)^{b_p} \quad t \in T, p \in P, s \in S \tag{9.18}$$

where a_p and b_p are parameters obtained by regression analysis, from historical or simulation data (Ballou, 1992). The inventory-throughput relationship (9.18) is illustrated in Figure 9.8. Note that, although it is assumed that $I_p()$ is independent of s, in practice it may be more appropriate to use a different function for each type of site (P-DC's, crossdocking centers, local DC's, etc.). Most network design models proposed in the literature do not take the *risk pooling* effects captured by function (9.18) into account: they assume either explicitly (Cohen and Moon, 1990; Arntzen et al., 1995; Dogan and Goetschalckx, 1999) or implicitly that the relationship between inventory levels and throughput is linear. If the historical throughput level and average inventory level observed for product p, in distribution center s, for the most recent season t, are $X^{I(0)}_{pst}$ and $I_p\left(X^{I(0)}_{pst}\right)$, respectively, then the ratio $X^{I(0)}_{pst}/I_p\left(X^{I(0)}_{pst}\right)$ is the familiar inventory turnover ratio, and its inverse

$$\rho^{(0)}_{pst} = I_p\left(X^{I(0)}_{pst}\right)/X^{I(0)}_{pst} \tag{9.19}$$

is the number of seasons of inventory kept in stock. Assuming that the relationship between inventory level and throughput is linear boils down to approximating $I_p\left(X^I_{pst}\right)$ by $\rho^{(0)}_{pst}X^I_{pst}$, as illustrated in Figure 9.8. Such an approximation may not be too bad in the vicinity of $X^{I(0)}_{pst}$, but the DC's throughputs are not known before the optimization model is solved and they can be far from historical values (mainly if a new DC is open or an existing DC is closed), which means that calculating inventory levels with historical inventory turnover ratios can be completely inadequate. An effort is therefore made in this paper to take risk pooling effects into account explicitly.

Function (9.18) provides the average inventory of product p required to support throughput X^I_{pst}. This quantity is needed to calculate inventory holding costs, but it cannot be used directly to calculate the space required to store the products in a warehouse because this space is proportional to maximum inventory levels and not to average inventory levels. For product p, the maximum level of cycle and safety stocks to be stored in a season is obtained by multiplying the average inventory level $I_p\left(X^I_{pst}\right)$ by an amplification factor β_p. In practice, the parameters $\beta_p,\ p \in P$, are estimated statistically from the company data on the inventory held in its facilities. From this it is seen that the throughputs and seasonal inventory levels in the DC's must respect the following storage space capacity constraints:

$$\sum_{p\in P_{ks}\cap MP} q_{pk} I^S_{pkst} + \sum_{p\in P_{ks}} q_{pk}\beta_p I_p(X^I_{pst}) \le \sum_{l\in L_{ks}} b_{lskt}Y_{ls} + \sum_{j\in J_{ks}} b_{jt}Z_j \quad t \in T, s \in S, k \in KS_s \tag{9.20}$$

The flows in all the facilities are also restricted by their receiving and shipping capacity. It is assumed here that this restriction can be properly expressed in terms of the total facility outflows, which leads to the following capacity constraints:

$$\sum_{p\in P_{ks}} q_{pk}\left(\sum_{n\in S^o_{ps}} F_{psnt} + \sum_{d\in D^o_{ps}}\sum_{i\in I_{psd}} F_{psdit}\right) \le \sum_{l\in L_{ks}} b_{lskt}Y_{ls} + \sum_{j\in J_{ks}} b_{jt}Z_j \quad t \in T, s \in S, k \in KW \tag{9.21}$$

Finally, the limited supply of raw materials and sub-assemblies which can be obtained from external vendors leads to the following inbound flow constraints:

$$\sum_{s \in S^o_{pv}} F_{pvst} \leq b_{pvt} \quad t \in T, p \in RM \cup SA, v \in V_p \qquad (9.22)$$

Modeling costs

The different costs and revenues associated with the arcs and nodes of a typical multinational logistics network are shown at the top of Figure 9.9, and their correspondence with the decision variables of the optimization model is indicated at the bottom of the figure. Note that several of the costs which are incurred in the network facilities are assigned to the models flow variables. For example, supply-order and receiving costs are assigned to inbound flow variables and customer-order, shipping as well as cycle and safety inventory holding costs are assigned to outbound flow variables. Note also that, in an international context, to take transfer prices and taxes into account correctly, it is necessary to derive an income statement for each network facility. This implies that certain costs associated with the network arcs must be split into the part paid by the origin and the part paid by the destination. For example, for arc (s, s') in Figure 9.9, the origin node pays the customer-order, shipping, transportation, inventory-in-transit and cycle/safety costs but the destination pays the supply-order and receiving costs. In addition, transfer prices are charged to node s' but they are a revenue for node s. Transportation costs are paid by the origin s but they are passed on to the destination s' and duties are paid by the destination. Note finally that, to compute after tax net revenues, the fixed selling costs of the selected markets and the fixed cost of support activities must also be taken into account.

The cost assignments described in Figure 9.9 are based on the following cost modeling assumptions:

- The prices and costs associated with the nodes of the network are given in local currency. The costs associated with the arcs of the network are given in source currency. Exchange rates are known and constant during the planning horizon considered.
- The fixed costs associated with facility layouts reflect potential changes of state (closing an existing facility, building or buying a new facility, changing the layout of a facility, etc.) and fixed

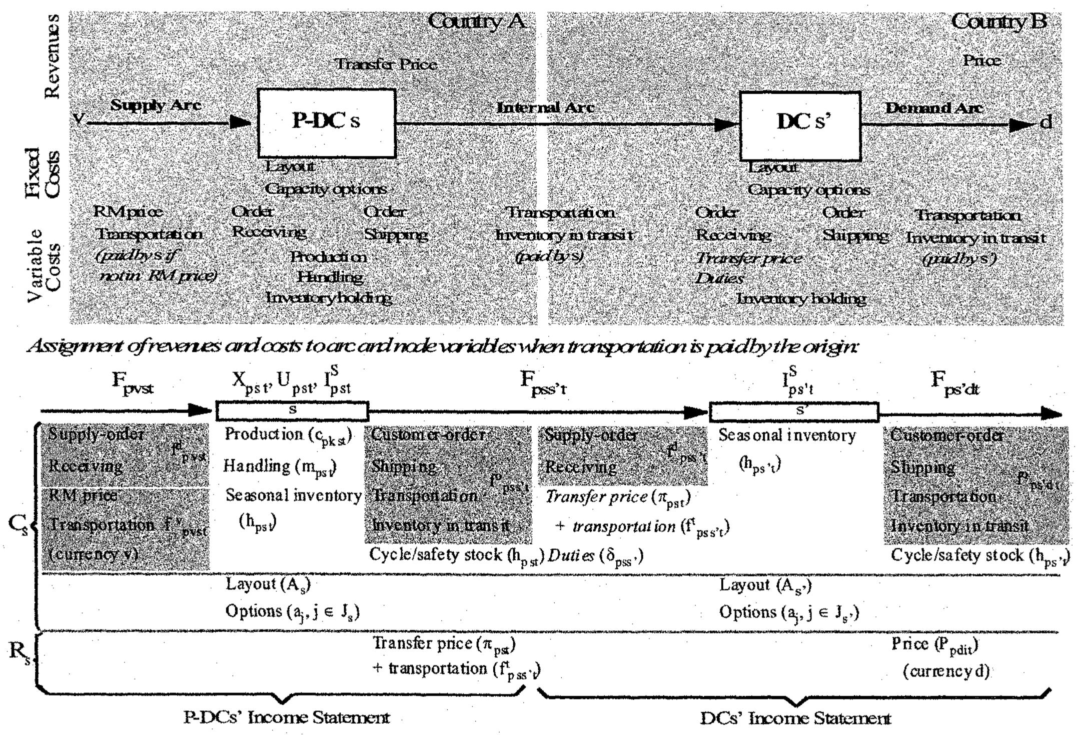

Figure 9.9. Mapping of costs and revenues on arcs and nodes.

operating expenditures, and they depend on the practical context of each potential node. The relevant fixed costs for different contexts are listed in Table 9.1. These costs are based on the engineering economy principles of capital recovery plus return over the planning horizon (Fabrycky and Torgersen, 1966). The fixed costs associated with potential capacity options also cover change of state and operating expenditures. The approach proposed to compute layout fixed costs can also be used to obtain capacity option fixed costs.

- Each time products cross a border; tariffs and duties are charged on the flow of merchandise and these are paid by the importer. In other words, these tariffs are calculated on the inflow to a given site from a foreign country of origin. These tariffs are based on the nature and class of the product. In the majority of countries, border tariffs are calculated on the CIF (Cost, Insurance and Freight) or the FOB (Free on Board) product value. In the model it is assumed that importers in all countries pay border tariffs based on CIF product values. To simplify the presentation, it is also assumed that there are no duty drawback or avoidance possibilities. An approach to model duty drawback and avoidance is presented by Arntzen et al. (1995).

- The transportation costs on the network arcs are paid by the origin. In practice, transportation costs usually display economies of scale with respect to shipment weight and distance, i.e., they can be modeled by a concave function $f(Q, d)$, where Q is the shipment weight and d is the distance between the origin and the destination. Different products can also be included in a given shipment. The flow on a network lane, say $\sum_p F_{pss't}$, corresponds to the sum of all the shipments made on arc (s, s') during season t. If the average weight of the shipments $Q_{ss't}$ on the arc is constant (e.g. truck load), then the shipment frequency during the season is given by $FR_{ss't} = \left(\sum_p \overline{w}_p F_{pss't}\right) / Q_{ss't}$ and the unit shipment cost $f^t_{pss't} = \overline{w}_p f(Q_{ss't}, d_{ss'})/Q_{ss't}$ is independent of the flow variables $F_{pss't}$. When this is the case, it is reasonable to assume that transportation costs are linear with respect to seasonal flows. On the other end, in practice, it is often the frequency of shipments

		Do not use site		*Use current layout ($l = 1$)*		*Use new layout ($l > 1$)*	
Initial state		Decision	Fixed cost (A_s^0)	Decision	Fixed cost (A_{1s})	Decision	Fixed cost (A_{ls})
Current facility	Owned	Close	-Closing cost	*Status-quo*	-Capital recovery -Opportunity cost -Operating cost	Change layout	-Setup cost -Capital recovery -Opportunity cost -Operating cost
	Rented	Close	-Closing cost -Lease penalty	*Status-quo*	-Rent -Operating cost	Change layout	-Setup cost -Rent -Operating cost
	Public	Stop	-Stopping cost	*Status-quo*	-Operating cost	Change layout	-Operating cost
Potential site	New facility or purchased & renovated	Do not use	-Zero			Build/ Buy	-Setup cost -Capital recovery -Opportunity cost -Operating cost
	Rented facility	Do not use	-Zero			Rent	-Setup cost -Rent -Operating cost
	Public	Do not use	-Zero			Use	-Starting cost -Operating cost

Table 9.1. Facility layout fixed costs in different contexts.

$FR_{ss't}$ that is considered constant. When this is the case, the average shipment weight is

$$Q_{ss't} = \left(\sum_p \overline{w}_p F_{pss't}\right) / FR_{ss't} \qquad (9.23)$$

and the unit shipment cost $f^t_{pss't}$ is a non-linear function of the seasonal flow variables. A successive linear programming approach to take these non-linearities into account is proposed by Fleischmann (1993). Another possible approach is to discretize the non-linear cost functions by introducing parallel arcs with different transportation costs and bounds on the flow variables. This approach, however, adds a large number of 0-1 variables. To simplify, in what follows, it is assumed that transportation costs are linear.

- Transfer prices for products sent in the internal network are fixed by the accounting department of the company and these do not include transportation costs from the source to the destination. In order to comply with laws and regulations, the transfer price of a given product shipped from a given source must be independent of its destination. In other words, the transfer price from the origin to the destination covers all the accumulated costs up to the shipping of the products from the origin and they include a predetermined margin. An approach to optimize transfer prices is presented by Vidal and Goetschalckx (2001).

- The income taxes paid in a country are calculated on the sum of the net revenues made by all facilities in this country. If a facility reports a loss, this loss is deducted from the total profit of the subsidiary before taxes. It is also assumed that the corporate taxes paid by the parent company are deferred until it pays dividends and that the decision to pay out dividends is independent of the design of the network. The parent company therefore only pays taxes on its local profits and it can be treated in the same way as the subsidiaries.

- The company wishes to maximize its after tax net revenues in a predetermined currency.

To calculate the total revenues and costs of a logistics network design, the following financial parameters and variables are required:

A_{ls} = Fixed cost of using layout l on site s for the planning horizon.

A_s^0 = Fixed cost of not using site s for the planning horizon.

a_j^1 = Fixed cost of using capacity option j for the planning horizon.

a_j^0 = Fixed cost of not using capacity option j for the planning horizon.

A_{mi}^M = Fixed selling cost incurred when marketing policy i is used for product-market m.

A_o^S = Fixed cost of support activities in country o.

c_{pkst} = Unit production cost of product p in production-distribution center s with technology $k \in KM_{ps}$ during season t.

m_{pst} = Unit handling cost for the transfer of product p to the stock of site s during season t.

f_{pvst}^v = Unit cost of the flow of product p between vendor v and site s during season t (this cost includes the product's price and the variable transportation cost).

f_{psnt}^o = Unit cost of the flow of product p between site s and node n paid by origin s during season t (this cost includes the customer-order processing cost, the shipping cost, the variable transportation cost and the inventory-in-transit holding cost).

f_{psnt}^t = Unit transportation cost of product p from site s to node n during season t (this cost is included in f_{psnt}^o).

f_{pnst}^d = Unit cost of the flow of product p between node n and site s paid by destination s during season t(this cost includes the supply-order processing costs and the receiving cost).

h_{pst} = Unit inventory holding cost of product p in facility s during season t.

π_{pst} = Transfer price of product p shipped from site s in season t.

$e_{oo'}$ = Exchange rate, i.e., number of units of country o currency by unit of country o' currency (the index $o = 0$ is given to the base currency, whether it is part of O or not).

δ_{pns} = Import duty rate applied to the CIF price of product p when transferred from the country of node n to the country of site s.

τ_o = Income tax rate of country o.

C_s = Total site s expenses for the planning horizon.

R_s = Total site s revenues for the planning horizon.

OP_o = Operating profit made in country o during the planning horizon.

OL_o = Operating loss made in country o during the planning horizon.

The revenues and expenses of the P-DCs and DCs, in local currency, are outlined in Table 9.2. The expression for the transfer costs of material inflows is obtained by first converting the transfer prices and transportation costs in local currency and then by adding the applicable duties. A similar approach is used to calculate other revenues and expenses.

Using the numbered elements of the expenditures and revenues in Table 9.2, it is seen that:

$$C_s = 1) + 2) + 3 + 5) + 6) + 7) + 9) + 10) \quad s \in Sd. \qquad (9.24)$$

$$C_s = 1) + 2) + 3 + 4) + 5) + 6) + 7) + 8) + 9) + 10) \quad s \in Spd. \qquad (9.25)$$

$$R_s = 11) + 12) \quad s \in S. \qquad (9.26)$$

The operating income for each national division is thus given by:

$$OI_o = \sum_{s \in S_o} (R_s - C_s) - \sum_{m \in M_o} \sum_{i \in I_m} A_{mi}^M Y_{mi} - A_o^S \quad o \in O \quad (9.27)$$

The corporate net revenues before taxes in the reference currency is given by the expression $\sum_{o \in O} e_{0o} OI_o$. To calculate corporate after tax profits, one must first separate the divisions where the margin is positive from the divisions where it is negative because there is no income tax to pay on losses. To do this, OI_o must be separated into its negative and positive parts by defining

$$\text{Operating Income } = OP_o - OL_o, \qquad (9.28)$$

where the operating profit $OP_o = OI_o$ if $OI_o > 0$ and the operating loss $OL_o = -OI_o$, otherwise. Clearly, for a given country, the operating profit OP_o and the operating loss OL_o cannot be simultaneously positive. Given this, it is seen that the after tax net revenues

		Distribution Center	Production-Distribution Center
Expenses	1) Inflows transfer cost	$\sum_{t\in T}\sum_{p\in P}\sum_{s'\in S^i_{ps}} (1+\delta_{ps's})e_{o(s),o(s')}(\pi_{ps't}+f^t_{ps'st})F_{ps'st}$	
	2) Raw materials	$\sum_{t\in T}\sum_{p\in RM\cup SA}\sum_{v\in V_p} (1+\delta_{pvs})e_{o(s),o(v)}f^v_{pvst}F_{pvst}$	
	3) Receptions from other sites	$\sum_{t\in T}\sum_{p\in P}\sum_{n\in V_p\cup S^i_{ps}} f^d_{pnst}F_{pnst}$	
	4) Production		$\sum_{t\in T}\sum_{p\in MP}\sum_{k\in KM_{ps}} c_{pkst}X_{pkst}$
	5) Facilities and options cost	$A^0_sY_{0s}+\sum_{l\in L_s} A_{ls}Y_{ls}+\sum_{j\in J_s}[a^1_jZ_j+a^0_j(1-Z_j)]$	
	6) Order cycle and safety stocks	$\sum_{t\in T}\sum_{p\in P} h_{pst}I_p(X^I_{pst})$	
	7) Seasonal stocks	$\sum_{t\in T}\sum_{p\in MP}\sum_{k\in KS_{ps}} h_{pst}I^S_{pkst}$	
	8) Handling		$\sum_{t\in T}\sum_{p\in MP} m_{pst}U_{pst}$
	9) Outflows to other sites	$\sum_{t\in T}\sum_{p\in P}\sum_{s'\in S^o_{ps}} f^o_{pss't}F_{pss't}$	
	10) Outflows to demand zones	$\sum_{t\in T}\sum_{p\in P}\sum_{d\in D^o_{ps}} f^o_{psdt}\sum_{i\in I_{psd}} F_{psdit}$	
Revenues	11) Outflows to other sites	$\sum_{t\in T}\sum_{p\in P}\sum_{s'\in S^o_{ps}} (\pi_{pst}+f^t_{pss't})F_{pss't}$	
	12) Outflows to demand zones	$\sum_{t\in T}\sum_{p\in P}\sum_{d\in D^o_{ps}} e_{o(s),o(d)}\sum_{i\in I_{psd}} P_{pdit}F_{psdit}$	

Table 9.2. Facilities expenses and revenues in local currency.

of the corporation in its reference currency is given by the expression $\sum_{o\in O} e_{0o}\,[(1-\tau_o)OP_o - OL_o]$.

3. Optimization model

Based on our previous discussion, the complete mathematical programming model proposed to optimize the structure of a global production-distribution network takes the following form:

$$Z = \max \sum_{o\in O} e_{0o}\,[(1-\tau_o)OP_o - OL_o] \qquad \text{(MIP)}$$

subject to:
- Demand and marketing policy constraints (9.1) and (9.2)
- Facility layout, space and exclusive options constraints (9.3), (9.6)

and (9.7)
- Upper bound on the number of DCs and P-DCs (9.4) and (9.5)
- Distribution centers throughput definition constraints (9.10) and (9.16)
- Production centers flow equilibrium constraints (9.8)
- Production facilities capacity constraints (9.11) and (9.12)
- Raw materials flow equilibrium constraints (9.13) and (9.14)
- Distribution centers seasonal inventory accounting constraints (9.9) and (9.15)
- Lower bounds on the distribution centers flow (9.17)
- Facilities storage capacity constraints (9.20)
- Facilities shipping (receiving) capacity constraints (9.21)
- External supply constraints (9.22)
- Definitions of the facility total cost (9.24) and (9.25)
- Definitions of facilities total revenue (9.26)
- Definitions of the national divisions operating income

$$\sum_{s\in S_o}(R_s - C_s) - \sum_{m\in M_o}\sum_{i\in I_m} A^M_{mi}Y^M_{mi} - OP_o + OL_o = A^S_o \quad o \in O \qquad (9.29)$$

-Non-negativity and binary constraints

$$Y^m_{mi} \in \{0,1\}, m \in M, i \in I_m; Y_{ls} \in \{0,1\}, s \in S, l \in L_s$$
$$Z_j \in \{0,1\}, j \in J$$
$$X_{pkst} \geq 0, \forall(p,k,s,t); U_{pst} \geq 0, \forall(p,s,t); I^S_{pkst} \geq 0, \forall(p,k,s,t);$$

$$F_{pnst} \geq 0, \forall(p,n,s,t); F_{psdit} \geq 0, \forall(p,s,d,i,t);$$
$$R_s \geq 0, C_s \geq 0, s \in S; OP_o \geq 0, OL_o \geq 0, o \in O.$$

This is a large scale non-linear mixed integer programming model. The non-linearities in the model are found in constraints (9.20), (9.24) and (9.25) and they all come from the inventory throughput functions. In order to solve the model efficiently, a method to cope with its size and its non-linearities must be used. Given the power of current MIP commercial solvers, the decision support system developed to generate and solve the model is based on a the solution of successive linear mixed-integer programming problems with a commercial solver, coupled with the use of valid inequalities (cuts) to strengthen the MIP formulation.

Experiments on the solution of particular cases of the model with Benders decomposition were made. It was found however that, to obtain good computation times with Benders decomposition, initial cuts had to be added to the model. It was also found that when these initial cuts were added to the model, the solution times obtained with CPLEX 8.1 were not worst than those obtained with Benders decomposition (Paquet et al., 2004). The approach used does not seek to obtain the global optimum: rather, it is perceived as a practical scenario improvement method based on reasonable approximations of the inventory-throughput functions.

An approach which could be used to linearize the problem is to replace $I_p\left(X_{pst}^I\right)$ by a piecewise linear approximation. This is equivalent to introducing alternative DC's at a given site with different lower and upper bounds on throughput, and adding an additional constraint on layout variables to ensure that only one of the alternative DC's can be used at each site. The problem with this approach is that it increases the number of 0-1 variables in the model significantly. This is why a successive MIP approach was developed. The approximation of the inventory throughput function used at iteration i of the solution method proposed is:

$$I_p\left(X_{pst}^I\right) = \rho_{pst}^{(i)} X_{pst}^I \tag{9.30}$$

where the slope $\rho_{pst}^{(i)}$ is calculated, at each iteration, from the flows of the last solution with the expression:

$$\rho_{pst}^{(i)} = I_p\left(X_{pst}^{I(i-1)}\right) / X_{pst}^{I(i-1)} = a_p\left(X_{pst}^{I(i-1)}\right)^{b_p-1} \tag{9.31}$$

The initial slope $\rho_{pst}^{(1)}$ is obtained by setting:

$$X_{pst}^{I(0)} = \left(\textstyle\sum_{d\in D} \underline{x}_{pdt}\right) / |S| \quad s \in S, p \in P, t \in T \tag{9.32}$$

or by using historical flows as in (9.19). Although the equal share flows obtained with (9.32) are not necessarily feasible, they yield an initial slope which can be used to start the procedure. An approach based on goal programming to arrive at feasible initial flows is proposed by Martel (2002). The iteration process is continued until the difference between the values of the objective function of two successive solutions is sufficiently small. The successive slope calculation process proposed

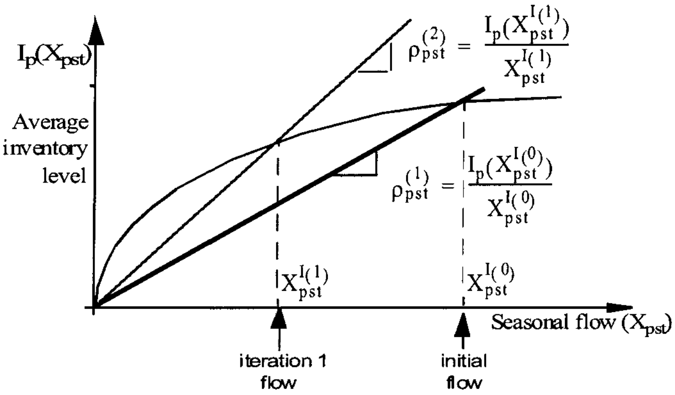

Figure 9.10. Successive linear approximations.

is illustrated in Figure 9.10. When the seasonal throughput obtained for center s during the i^{th} iteration $\left(X^{I(i)}_{pst}\right)$ is positive, the slope can be calculated using relation (9.31). When $X^{I(i)}_{pst} = 0$, however, which necessarily occurs when a site is not used, the slope is not revised and the value obtained at the preceding iteration is retained. A heuristic approach similar to ours is used by Kim and Pardalos (2000) to solve concave piecewise linear network flow problems. To describe the solution algorithm formally, the following notation is needed:

MIP(i) = The mathematical program obtained by replacing $I_p\left(X^I_{pst}\right)$ in the constraints (9.20), (9.23) and (9.24) of MIP by $\rho^{(i)}_{pst}X^I_{pst}$ and by adding appropriate initial cuts.

Sol_i = The solution obtained by solving MIP(i).

$Z_i(Sol)$ = The value of the objective function of MIP(i) for solution Sol.

$Z(i)$ = The exact value of Sol_i, i.e., the value obtained by using the site cost definitions (9.24) and (9.25) to evaluate Sol_i.

Note that, because of the nature of the approximation made, we have:

$$Z(i) = Z_{i+1}(Sol_i)$$

The algorithm used to initialize the solution process and to improve the solutions obtained iteratively is the following:

1) *Initialization:*
   Set $i = 0$
   Obtain *equal-share* initial throughputs for the centers by computing:
   $X_{pst}^{I(0)} = \left(\sum_{d \in D} \underline{x}_{pdt}\right) / |S| \; s \in S, p \in P, t \in T$
2) *Linearization with the last iteration throughputs.*
   $i = i + 1$
   For each product p, each center $s \in S$ and each season $t \in$ T,
   If the throughput $X_{pst}^{I(i-1)}$ is positive, compute the revised inventory duration with:
   $\rho_{pst}^{(i)} = I_p\left(X_{pst}^{I(i-1)}\right) / X_{pst}^{I(i-1)}$
   If the throughput $X_{pst}^{I(i-1)}$ is null, keep the inventory duration used at the previous iteration.
   If $i > 1$, set $Z(i-1) = Z_i(Sol_{i-1})$
3) *Check the stopping condition.*
   If $\{i > 2\}$ and $\{[Z(i-1) - Z(i-2)]/Z(i-2) < \epsilon\}$, end.
4) *Solve the mixed-integer programming problem.*
   Find the solution Sol_i of MIP(i)
   Go back to Step 2,

where ϵ is an acceptable tolerance.

Note that if relation (9.23) is added to the model, the solution approach proposed can easily be modified to take concave transportation costs into account. Also, instead of using inventory durations to approximate the inventory-throughput functions, it is possible to use the gradient of $I_p\left(X_{pst}^{I}\right)$ evaluated at $X_{pst}^{I(i-1)}$ and to limit the throughput change at iteration i to a trust region around $X_{pst}^{I(i-1)}$. This approach, proposed by Martel and Vankatadri (1999), provides a better approximation but it is more difficult to implement and less intuitive. The solution approach proposed here has given very satisfactory results in several real life projects. It was used, for example, to reengineer the North-American production-distribution network of Domtar, one of the largest fine paper producers in the world. The project involved the consideration of 12 paper mills, 13 conversion sub-contractors and 50 distribution centers. More than 100 product families and 1 000 demand zones were taken into account. The problems to solve had about 300 000 variables, including 75 binary variables.

4. Conclusion

This text proposes a mathematical programming approach to design international production-distribution networks for make-to-stock products with convergent manufacturing processes. A more general version of the model proposed and the solution method described were implemented in a commercial supply chain design tool which is now available on the market. The tool was used to solve several real life logistics network design problems. Work is currently in progress to expand the approach to make-to-order contexts and to divergent manufacturing process industries.

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