

1-

A single server queueing system is known to have Poisson arrivals and exponential service times. However, the arrival rate and service time are state dependent. As the queue becomes longer, servers work faster, and the arrival rate declines, yielding the following functions:

$$\lambda_0 = 5/\text{hour} \quad \lambda_1 = 3/\text{hour} \quad \lambda_2 = 2/\text{hour} \quad \lambda_n = 0/\text{hour} \quad n \geq 3$$

$$\mu_0 = 0/\text{hour} \quad \mu_1 = 2/\text{hour} \quad \mu_2 = 3/\text{hour} \quad \mu_n = 4/\text{hour} \quad n \geq 3$$

- Calculate the state probabilities.
- What are the average arrival and service rates?
- Calculate L_q, L_s, W_q, W_s .

Answer:

2-

An $M/M/1/\infty$, FCFS, queue has an arrival rate of 100/hour and a service rate of 140/hour.

- Write the equation for P_n , the probability of being in state n .
- Calculate L_q, L_s, W_q , and W_s .
- What is the probability that a customer's waiting time in queue is less than 3 minutes?
- What is the probability that a customer's waiting time in system is less than 3 minutes?
- Compare your answer to part b to the solution to Prob. 15 of Chap. 4. Does your answer make sense, if both queues are $M/M/1/\infty$?

Answer :

3-

After observing a single server queue for several days, the following steady-state probabilities have been determined: $P_0 = .4, P_1 = .3, P_2 = .2, P_3 = .05, P_4 = .05$. The arrival rate is 10 customers/hour.

- Determine L_s and L_q .

Arrival Rate is 10 customer /hour

Answer :

Repeat Prob. 3, with state probabilities $P_0 = .2, P_1 = .4, P_2 = .2, P_3 = .2$ ($\lambda = 10/\text{hour}$), with two servers.

Answer :

4-

A queueing system receives Poisson arrivals at the rate of 5/hour. The single server has a uniform service time distribution, with a range from 4 minutes to 6 minutes. Determine L_q, L_s, W_q , and W_s . (Hint: Properties of the uniform distribution are given in Chap. 3.)

Answer:

5-

An airline phone reservation line has one server, and a buffer for two customers. The arrival rate is 6 customers/hour, and the service rate is just 5 customers/hour. Arrivals are Poisson and service times are exponential.

- (a) Estimate L_q and the average number of customers served per hour.
- (b) From Fig. 5.4, estimate L_q for a buffer of size 5. In words, what is the impact of the increased buffer size on the number of customers served per hour?
- (c) Repeat part a for a buffer of size 2, with two servers.
- (d) If employees are paid \$15/hour, how large would the profit per customer have to be to justify adding a second server (with a buffer of size 2)?

Answer: